COUPLING LOSSES IN SUPERCONDUCTING CAVITIES*

H. Hahn and J.M. Miller†
Brookhaven National Laboratory
Upton, New York

I. INTRODUCTION

The nature of the residual losses in superconducting cavities is not yet completely understood. Magnetic flux trapped in the superconducting surface is known to be a source of losses,[1] which can, in practice, be eliminated by proper magnetic shielding. On the other hand, it was shown that losses in the coupling mechanism can, under certain conditions, represent an important contribution.[2]

In this paper we intend to investigate theoretically the losses of a piston-type coupling mechanism which consists of a coaxial cable with a coupling loop and a below-cutoff tube coupled to the TE_{011} cavity through an iris. The coupling to the cavity by means of waveguides can be made practically lossless and need not be considered here. It was possible to derive expressions for the maximum measurable quality factor which suggest ways of reducing the residual losses due to the coupling mechanism. Previously published Q measurements[3] are analyzed in regard to coupling losses and it is concluded that the residual losses must have a different origin.

II. THE UNLOADED QUALITY FACTOR

The unloaded quality factor (no radiation losses) of a cavity is defined by $Q_0^{-1} = P/\omega W$, where P represents the wall losses, W the total stored energy, and ω the resonant frequency. In the case of a cavity with a homogeneous surface, Q_0 is related to the surface impedance by $Q_0 R = G$, where R is the surface impedance, and the geometrical constant, G, is given in natural units by

$$G^{-1} = \frac{\int \vec{B}^* \cdot \vec{B}\, dS}{k \int \vec{B}^* \cdot \vec{B}\, dV} \tag{1}$$

with the wave number $k = \omega$.

At first we will derive G of the TE_{011} cavity coupled to an infinitely long below-cutoff waveguide by an iris in one end plate. The iris is located at the point of maximum magnetic field of the TE_{011} cavity and on the axis of the TE_{11} guide (Fig. 1).

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†Summer student from Queens College of the City University of New York.

1. J.M. Pierce, H.A. Schwettman, W.M. Fairbank, and P.B. Wilson, in Proc. 9th Intern. Conf. Low Temperature Physics, Columbus, Ohio (Plenum Press, New York, 1965), p.396.
2. H. Schopper, to be published in the Proc. of the 1968 Linear Accelerator Conference, Brookhaven.
3. H. Hahn, H.J. Halama, and E.H. Foster, J. Appl. Phys. 39, 2606 (1968). See also these Proceedings, p. 13.

Homogeneous surface properties are assumed, which means that both the cavity and the below-cutoff tube must be lead-plated.

The expressions for the magnetic field in the unperturbed TE_{011} cavity (region I) can be written in circular cylinder coordinates as

$$
\begin{aligned}
B_r^I &= J_1\,(j'_{01}\ r/a)\ \cos \pi z/\ell \\
B_\theta^I &= 0 \\
B_z^I &= \frac{j'_{01}}{\pi}\frac{\ell}{a}\ J_0\,(j'_{01}\ r/a)\ \sin \pi z/\ell \quad ,
\end{aligned}
\tag{2}
$$

where a is the radius and ℓ the length of the cavity. The following integrals will be needed to evaluate Eq. (1):

$$\int \vec{B}^{I*} \cdot \vec{B}^I\, dV = \frac{\pi}{2}\, a^2\ \ell\ J_0^2\ (j'_{01})(1 + g) \tag{3}$$

$$\int \vec{B}^{I*} \cdot \vec{B}^I\, dS = \pi\ a\ \ell\ J_0^2\ (j'_{01})(2a/\ell + g) \quad , \tag{4}$$

where

$$g = \left(\frac{j'_{01}}{\pi}\frac{\ell}{a}\right)^2 \quad .$$

The geometrical constant in the absence of a coupling hole follows as

$$G^{-1} = \frac{4}{k\ell}\,\frac{1 + \frac{1}{2}\, g\ell/a}{1 + g} \quad . \tag{5}$$

The resonant frequency in the unperturbed case is found from

$$k\ell = \pi(1 + g)^{\frac{1}{2}} \quad ,$$

and finally

$$G^{-1} = \frac{4}{\pi}\,\frac{1 + \frac{1}{2}\, g\ell/a}{(1 + g)^{3/2}} \quad . \tag{6}$$

In the Brookhaven cavity,[3] $\ell = 2a$ and $G^{-1} = 0.483$.

Due to the iris of radius ρ, a TE_{11} far-field will be excited in the below-cutoff tube with radius b which can be approximated for $z > 0$ by

$$
\begin{aligned}
B_r^{II} &\approx C\ J'_1(\alpha r)\ e^{-\alpha z}\ ie^{-i\theta} \\
B_\theta^{II} &\approx C\ \frac{J_1(\alpha r)}{\alpha r}\ e^{-\alpha z}\ \ e^{-i\theta} \\
B_z^{II} &\approx -\ C\ J_1(\alpha r)\ e^{-\alpha z}\ ie^{-i\theta} \quad ,
\end{aligned}
\tag{7}
$$

where the attenuation constant $\alpha^2 = (j'_{11}/b)^2 - k^2$. In practically important cases where the cutoff tube has a radius $b \ll a$, α is simply $\alpha \approx j'_{11}/b$. The amplitude C follows from Bethe's small-hole approximation,[4]

$$C = \frac{4}{3\pi} \frac{j_{11}^{\prime 3}}{(j_{11}^{\prime 2} - 1)\, J_1(j_{11}^{\prime})} \left(\frac{\rho}{b} \right)^3 \tag{8}$$

or $C = 1.90\, (\rho/b)^3$. This result can be verified in the case of $\rho = b$, for which direct field matching leads to $C = 2\, j_{11}^{\prime}\, (j_{11}^{\prime 2} - 1)^{-1} = 1.54$.

To evaluate the geometrical constant in the presence of an iris, we will need the following integrals taken over the infinitely long unperturbed (i.e. open) below-cutoff tube:

$$\int \vec{B}^{II*} \cdot \vec{B}^{II}\, dV = \frac{\pi}{2}\, b^3 \frac{(j_{11}^{\prime 2} - 1)\, J_1^2(j_{11}^{\prime})}{j_{11}^{\prime 3}}\, C^2 \tag{9}$$

and

$$\int \vec{B}^{II*} \cdot \vec{B}^{II}\, dS = \frac{\pi}{2}\, b^2 \frac{J_1^2(j_{11}^{\prime})}{j_{11}^{\prime 3}}\, (1 + j_{11}^{\prime 2})\, C^2 \quad . \tag{10}$$

Because our analysis neglects near-fields in the vicinity of the iris, Eqs. (9) and (10) are obviously only approximate expressions. This is, however, inconsequential since the geometrical constant of the cavity is practically unaffected by the cutoff tube. To show this, we compare Eqs. (3) and (4) with Eqs. (9) and (10) and find that for $\ell = 2a$

$$\frac{\int \vec{B}^{II*} \cdot \vec{B}^{II}\, dV}{\int \vec{B}^{I*} \cdot \vec{B}^{I}\, dV} \approx 0.2 \left(\frac{b}{a} \right)^3 \left(\frac{\rho}{b} \right)^6$$

and

$$\frac{\int \vec{B}^{II*} \cdot \vec{B}^{II}\, dS}{\int \vec{B}^{I*} \cdot \vec{B}^{I}\, dS} \approx 0.2 \left(\frac{b}{a} \right)^2 \left(\frac{\rho}{b} \right)^6 \quad .$$

We conclude that the geometrical constant of the Brookhaven cavity ($b/a \approx 0.1$) is completely determined by the unperturbed TE_{011} cavity and is given by Eq. (6). Thus the losses of a lead-plated below-cutoff tube are negligible.

The situation changes completely if a coaxial cable with a coupling loop is inserted into the below-cutoff tube. This exposes an area A_1 of normal metal, of which the cable is made, to a magnetic field resulting in additional rf losses:

$$\Delta P \approx \tfrac{1}{2}\, R_N\, A_1\, C^2\, e^{-2\alpha z} \quad , \tag{11}$$

where R_N is the surface resistance in the normal state. The geometrical constant is now dependent on the loop position according to

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4. R.E. Collin, Foundations for Microwave Engineering (McGraw-Hill, New York, 1966), p. 190.

$$G^{-1} = \frac{\int \vec{B}^{I*} \cdot \vec{B}^{I}\, dS + (R_N/R_S)\, A_1\, C^2\, e^{-2\alpha z}}{k \int \vec{B}^{I*} \cdot \vec{B}^{I}\, dV} \quad . \tag{12}$$

At low temperatures the ratio of normal to superconducting surface resistance, R_N/R_S, becomes very large and the influence of the coupling loop may not be negligible. A method of eliminating the cable influence by measuring at various loop positions was described by Schopper.[2] This method is rather time consuming and we will investigate in the following section whether the influence of the coupling loop can be eliminated or at least reduced.

III. THE MAXIMUM MEASURABLE $\hat{Q}_o$

Equation (12) suggests that we may reduce the coupling losses by pulling back the coupling loop from the cavity. This does, however, limit the power which may be coupled out of the cavity, the lower limit being determined by the sensitivity of the instrument used for the rf power measurement. The maximum unloaded quality factor measurable with a given experimental setup is (in natural units) obviously

$$\hat{Q}_o = \frac{kW}{\Delta P} \quad . \tag{13}$$

ΔP is determined by the minimum power detectable, P_{min}, because the radiated power is related to ΔP by the loop geometry. Neglecting the inductance[5] and resistance of the loop, the power radiated into the cable is obtained as

$$P_{rad} = \omega^2\, A_2^2\, C^2\, e^{-2\alpha z} / Z \quad , \tag{14}$$

where Z is the characteristic impedance of the cable and A_2 the area of the loop. It follows that

$$\hat{Q}_o = \frac{2\, k^3\, W}{R_N\, Z} \frac{A_2^2}{A_1} \frac{1}{P_{min}} \quad . \tag{15}$$

We see that $\hat{Q}_o$ can be influenced by the geometry of the coupling loop and by the sensitivity of the power meter. It seems advantageous to work with relatively large loops (reactance about equal to Z) and to use a low noise, high gain amplifier at the cable end.

On the other hand, it is worthwhile to point out that the coupling loop hole size ρ is, in principle, irrelevant to the problem; it determines only the position of the loop at which the measurement is made and consequently the required length of the below-cutoff tube. Because of difficulties with the plating of the superconducting surfaces, the smallest coupling hole is nevertheless desirable.

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5. The self-inductance of the loop is about equal to its diameter. C.C. Johnson, Field and Wave Electrodynamics (McGraw-Hill, New York, 1965), p. 202. See also S. Ramo, J.R. Whinnery, and T. van Duzer, Fields and Waves in Communication Electronics (John Wiley & Sons, New York, 1965), p. 311.

The Q measurements reported in Refs. 3 and 6 were performed with the same coupling loop. In Ref. 3, the iris hole radius was reduced, and a TWT amplifier with 30 dB gain was inserted between cable output and diode. From the preceding discussion it follows that the maximum measurable $\hat{Q}_o$ was increased in Ref. 3 by a factor of 1000 resulting in a $\hat{Q}_o \geq 10^{12}$. This is much larger than the measured quality factor which permits the conclusion that the residual resistance quoted in Ref. 3 was not caused by position-dependent losses in the coupling mechanism. On the other hand, recent measurements on another cavity of similar geometry show clear evidence of position-independent coupling losses. The analysis of this type of loss is not covered in this paper, and it is not impossible that the residual Q in Ref. 3 is caused by position-independent losses.

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6. H. Hahn, H.J. Halama, and E.H. Foster, in Proc. 6th Intern. Conf. High Energy Accelerators, Cambridge, Mass., 1967, p. A-139.

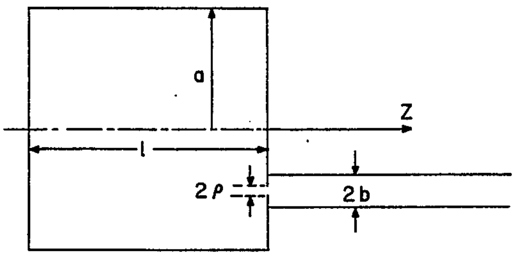

Fig. 1. Geometry of TE_{011} cavity with below-cutoff tube.