APPLICATIONS OF THE FOUNTAIN EFFECT IN SUPERFLUID HELIUM*

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INTRODUCTION

If large cryogenic systems using superfluid helium are to be completely practical, simple and efficient methods must be available 1) for providing steady-state refrigeration from a central plant to an individual experiment regardless of its elevation, and 2) for transferring liquid helium in its superfluid state from one reservoir to another. The methods whereby these operations would be accomplished with ordinary liquid helium will not work with superfluid helium. However, the thermomechanical effect, or fountain effect, in superfluid helium can be utilized in the construction of a fountain pressure pump to accomplish both of these objectives.

Consider two superfluid reservoirs connected by a "superleak." If one reservoir is at temperature T and the other at T + ΔT, then in equilibrium there exists a large hydrostatic pressure difference ΔP, known as the fountain pressure, which is given by:

$$\frac{\Delta P}{\Delta T} = \rho S \quad , \tag{1}$$

where ρ is the density and S is the entropy per gram of liquid helium. Observation of this thermomechanical effect was first reported by Allen and Jones[1] in 1938, and the equation relating its magnitude to the density and entropy of the liquid was derived by H. London[2] in 1939. The magnitude of the fountain pressure is surprisingly large. The hydrostatic pressure difference can be as large as 500 mm of Hg, an order of magnitude greater than the vapor pressure of helium at the lambda point. Combined with the low density of liquid helium, this pressure is sufficient to support a column of helium in excess of 100 ft in height. The thermomechanical effect thus provides the means for constructing a fountain pressure pump that can be used both to provide steady-state refrigeration to elevated experiments and to transfer superfluid helium from one reservoir to another.

THE IDEAL FOUNTAIN PRESSURE PUMP

For large temperature differences the equation for the fountain pressure [Eq. (1)] must be integrated, since the entropy of helium II is a very strong function of temperature between 1.2°K and 2.1°K.[3] The integrated equation is

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1. J.F. Allen and H. Jones, Nature <u>141</u>, 243 (1938).
2. H. London, Proc. Roy. Soc. <u>A171</u>, 484 (1939).
3. F. London, <u>Superfluids</u> (Dover Publications, Inc., New York, 1954), Vol. II. p. 75.

$$P_2 - P_1 = \int_{T_1}^{T_2} \rho(T)\ S(T)\ dT \quad , \tag{2}$$

where P_1 and P_2 are the pressures on each side of the reservoir, T_1 and T_2 are the corresponding temperatures, ρ is the density of helium, and S in the entropy per gram of helium. This relation has been found to hold within experimental errors up to a temperature difference of 0.6°K and a pressure difference of 500 mm Hg.[4] That pressure difference corresponds to a column of liquid helium 46.8 m in height.

A schematic drawing indicating how the fountain pressure pump could be used to provide refrigeration to an elevated experiment is shown in Fig. 1. If the two reservoirs are at different temperatures T_1 and T_2, then the pressure difference across the superleak will be

$$P_2 - P_1 = \rho gh + P_v(T_2) - P_v(T_1) \quad , \tag{3}$$

where ρ is the density of helium, g is acceleration due to gravity, h is the difference in height between the two levels, and $P_v(T)$ is the vapor pressure of helium as a function of temperature. Solving for h and using Eq. (2) yields

$$h(T_1,T_2) = \frac{1}{\rho g} \left(\int_{T_1}^{T_2} \rho S dT + P_v(T_1) - P_v(T_2) \right) \quad . \tag{4}$$

In Fig. 2 this height is plotted as a function of T_2 for T_1 = 1.85°K.

Now consider the effectiveness of the fountain pressure pump in supplying steady-state refrigeration to an elevated reservoir as illustrated in Fig. 1. For a heat input $\dot{Q}_T$ in the upper reservoir the central refrigerator must provide helium to the lower reservoir at the rate $\dot{m}_T$. Part of this helium, $\dot{m}_2$, flows through the superleak and evaporates from the upper reservoir at temperature T_2, and part, $\dot{m}_1$, evaporates directly from the lower reservoir at temperature T_1. The ratio of $\dot{m}_1$ to $\dot{m}_2$ is determined by the condition that the heat generated in the lower reservoir by the helium flowing through the superleak must equal the heat removed by vaporization. According to the mechanocaloric effect[5] we have

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{S(T_1)T_1}{L(T_1)} \quad , \tag{5}$$

where T_1 is the temperature of the lower reservoir, and $S(T_1)$ and $L(T_1)$ are the entropy and latent heat of vaporization for helium at temperature T_1. Using Eq. (5), the helium used is

$$\dot{m}_T = \dot{m}_1 + \dot{m}_2 = \dot{m}_2 \left(1 + \frac{S(T_1)T_1}{L(T_1)} \right) \quad . \tag{6}$$

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4. E.F. Hammel, Jr. and W.E. Keller, Phys. Rev. 124, 1641 (1961).
5. F. London, op. cit., Vol. II, p. 69.

The heat input $\dot{Q}_T$ in the upper reservoir is absorbed in part by the mechanocaloric effect of superfluid flowing through the fountain pressure pump and, in part, by evaporation of helium at the upper reservoir. Thus we have

$$\dot{Q}_T = \dot{m}_2 L(T_2) + \dot{m}_2 S(T_2) T_2 \quad . \tag{7}$$

The alternative to using a fountain pressure pump to deliver helium to the upper reservoir from a central refrigerator would be to build a separate refrigerator at the elevated position. To compare the two possible systems, it is useful to consider the ratio

$$R(T_1,T_2) = \frac{\dfrac{T_1}{T_2}\dfrac{\dot{Q}_T}{\dot{m}_T}}{\dfrac{\dot{Q}_r}{\dot{m}_r}} = \frac{T_1}{T_2} \times \frac{1 + \dfrac{S(T_2)T_2}{L(T_2)}}{1 + \dfrac{S(T_1)T_1}{L(T_1)}} \quad , \tag{8}$$

where $\dot{Q}_r = L(T_2)\dot{m}_r$ is the cooling supplied by a separate elevated refrigerator. $\dot{Q}_T/\dot{m}_T$ and $\dot{Q}_r/\dot{m}_r$ give the ratios of refrigeration to helium evaporation for the fountain pressure pump system and the separate elevated refrigerator respectively, while the factor T_1/T_2 reflects the different temperatures of helium supplied by the two refrigeration systems. The ratio $R(T_1,T_2)$ is plotted as a function of T_2 in Fig. 3.

The fountain pressure pump can also be used to transfer superfluid helium from one reservoir to another. The superleak only allows the superfluid component to be transferred so that again the mechanocaloric effect is operative. If liquid is transferred at a rate $\dot{m}_2$, then heat must be removed from the lower reservoir at a rate

$$\dot{Q}_1 = \dot{m}_2 S(T_1) T_1$$

and heat must be added to the upper reservoir at a rate

$$\dot{Q}_2 = \dot{m}_2 S(T_2) T_2 \quad .$$

If heat is removed from the lower reservoir by evaporating part of the helium, then the evaporation rate must be

$$\dot{m}_1 = \frac{\dot{Q}_1}{L(T_1)} = \dot{m}_2 \frac{S(T_1)T_1}{L(T_1)} \quad .$$

The ratio of helium lost to helium transferred is just

$$\frac{\dot{m}_1}{\dot{m}_2} = \frac{S(T_1)T_1}{L(T_1)} = 5\% \quad \text{at } T_1 = 1.85^{\circ}\mathrm{K} \quad .$$

SPECIAL PROBLEMS

In general, a superleak is any passage that permits the flow of the superfluid component of helium II, but prevents the flow of the normal component. For small flow rates the superfluid component can flow through the superleak without a pressure gradient. For flow rates above the superfluid critical velocity, v_{sc}, various dissipative mechanisms appear. The critical velocity depends on the size of the channel through which the helium flows and has been found to be as high as 20 cm/sec in some very narrow channels.[6] For large flow rates one very simple type of superleak is a powder-filled plug. It is difficult to compute a critical velocity in the case of a powder superleak since there is no way of knowing the distribution of pore sizes. However, experiments have shown that tubes packed with powder can maintain a flow rate of 1.2 liter/min·cm^2 of tube cross section.[7] Thus a powder superleak one inch in diameter could transport 360 liters per hour.

Care must be exercised so that there is not appreciable conduction of heat across the superleak. Heat can flow by conduction through the walls of the tube, but this can be minimized by using tubes with low conductivity. The normal fluid must be tightly clamped since any back flow of normal fluid would carry heat. There will also be heat conduction by diffusion through the clamped fluid but this should be negligible.

In our previous discussion of the fountain pressure pump (see Fig. 1), where it is used to provide refrigeration to the upper reservoir, we have assumed that the temperature of the upper reservoir is equal to the temperature just above the superleak. Actually there will be a small temperature gradient along the tube due to the large flow rates involved in applications of interest. In steady-state operation the normal fluid will be stationary while the superfluid will have an upward velocity. The flow rate of the superfluid will exceed its critical velocity since as the size of the tube increases, the critical velocity approaches zero.[8] Under these conditions a mutual friction force is operative and must be included in the calculation of the flow. The equations governing the flow are

$$\rho_s \frac{d\vec{v}_s}{dt} = \rho_s S\nabla T - \frac{\rho_s}{\rho} \nabla P - \vec{F}_{ns} - \rho_s \nabla\Phi \tag{10}$$

and

$$\rho_n \frac{d\vec{v}_n}{dt} = - \rho_s S\nabla T - \frac{\rho_n}{\rho} \nabla P + \vec{F}_{ns} + \eta_n \nabla^2 \vec{v}_n - \rho_n \nabla\Phi \quad , \tag{11}$$

where ρ_s = superfluid density,
ρ_n = normal fluid density,
$\vec{v}_s$ = superfluid velocity,
$\vec{v}_n$ = normal fluid velocity,
η_n = normal fluid viscosity,
Φ = gravitational potential, and
$\vec{F}_{ns}$ = mutual friction force.[9,10]

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6. W.M. van Alphen, J.F. Olijhoek, and R. de Bruyn Ouboter, Superfluid Helium (Academic Press, New York, 1966), p. 187.
7. Ibid., p. 190.
8. J. Wilks, Properties of Liquid and Solid Helium (Clarendon Press, Oxford, 1967), p. 390.
9. J. Wilks, op. cit., p. 377.
10. F. London, op. cit., Vol. II, p. 83.

For use in the fountain pressure pump these equations simplify since

$$\frac{d\vec{v}_s}{dt} = \frac{d\vec{v}_n}{dt} = \vec{v}_n = 0 \quad .$$

Adding Eqs. (10) and (11) gives

$$\nabla P = - \rho \nabla \Phi \tag{12}$$

and putting this into Eq. (10) yields

$$\rho_s S \nabla T = \vec{F}_{ns} \quad . \tag{13}$$

The mutual friction force has the form[11]

$$\vec{F}_{ns} = A \rho_s \rho_n (v_n - v_s)^2 \, (\vec{v}_n - \vec{v}_s) \quad ,$$

where A is an empirical constant which is a slowly varying function of temperature and geometry. Evaluating Eq. (13) for $T = 1.9^{o}K$ and using $A = 70$ cm·sec/g,[12] one finds

$$\nabla T = \frac{6.1 \times 10^{-7} \, v_s^3}{(\mathrm{cm/sec})^3} \, \frac{^{o}K}{\mathrm{cm}} \quad .$$

If we require that the ΔT in a column of helium 10 m in height be less than $0.01^{o}K$, then we must have $\vec{v}_s \leq 2.5$ cm/sec. This means that a one-inch diameter column of helium could support the flow of 46 liters of superfluid per hour which represents $\simeq$ 45 W of cooling.

The flow velocity can be considerably higher in a system which uses the fountain pressure pump for transfer of liquid helium. In the transfer system, heat can be supplied just above the superleak so that the flow of helium up the tube consists of both normal and superfluid components. Ideally, $\vec{v}_n = \vec{v}_s$ so that the mutual friction force is zero. Under these conditions calculations show that the temperature gradient along the tube is negligible.

The fountain pressure pump appears to offer a simple and efficient way to provide cooling a distance from a central refrigerator as well as a means for transfer of liquid helium below the lambda point. Experiments are in progress to demonstrate the usefulness of the fountain pressure pump in large-scale systems.

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11. J. Wilks, op. cit., p. 381.

12. H.C. Kramers, Superfluid Helium (Academic Press, New York, 1966), p. 205.

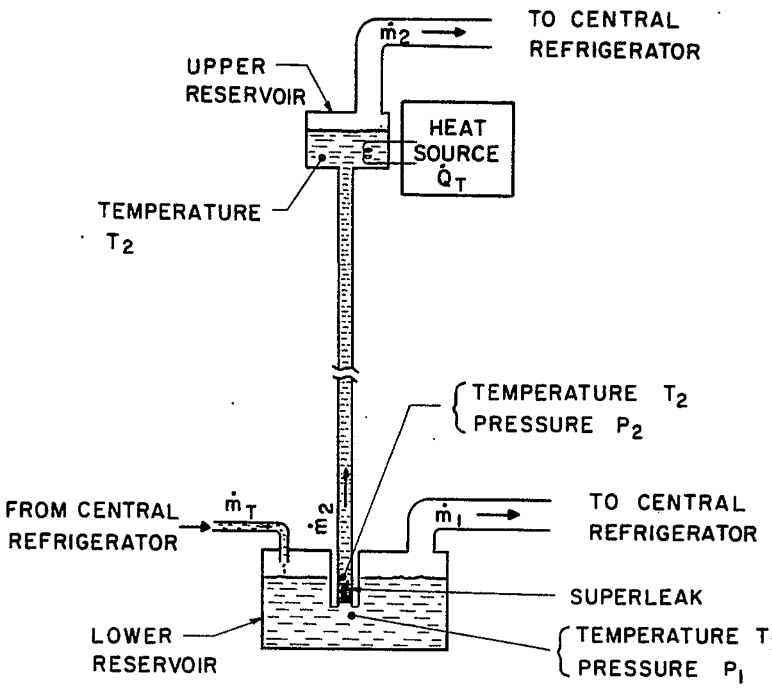

Fig. 1. Schematic drawing of a fountain pressure pump used to provide refrigeration.

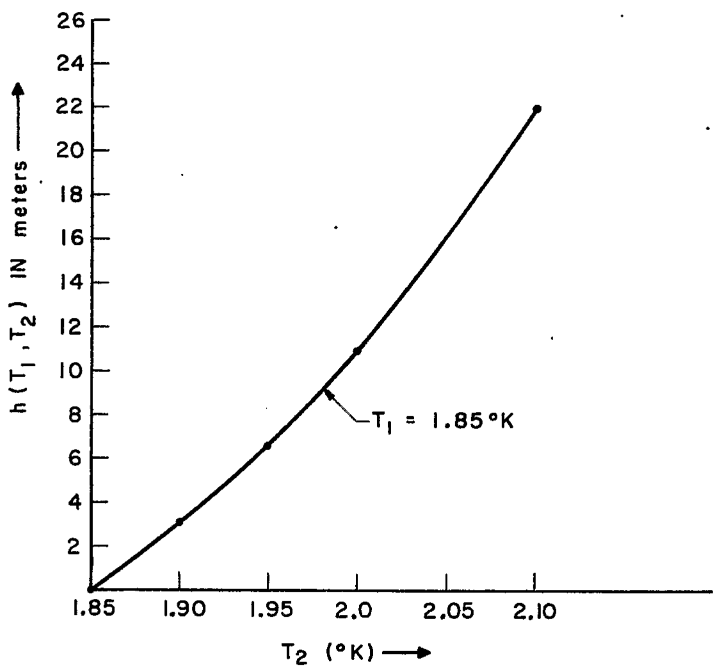

Fig. 2. The difference in height $h(T_1,T_2)$ as a function of T_2 with $T_1 = 1.85^oK$.

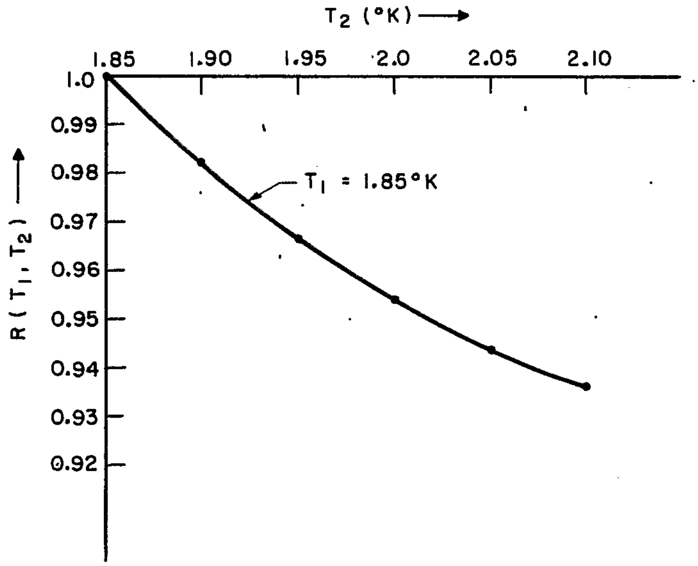

Fig. 3. The ratio $R(T_1,T_2)$ as a function of T_2 for $T_1 = 1.85^oK$.