CRITICAL CURRENT BEHAVIOR OF HARD SUPERCONDUCTORS

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I. INTRODUCTION

For many applications it is not enough that a superconductor have a high critical temperature T_c and high upper critical magnetic field H_{c2}; often a high critical current density $J_c(B,T)$ in high magnetic fields and at reasonable temperatures is also essential. High critical current density in the presence of a transverse magnetic field is not an intrinsic property of superconductors. Instead the critical current density in the presence of a transverse field is a nonequilibrium property that is closely analogous to the permanent magnetism of a hard ferromagnet, hence the designation hard superconductor. Just as the coercive force of a permanent ferromagnet depends on the strength of pinning of ferromagnetic domain walls, the critical current densities, or corresponding field gradients, in hard superconductors depend on pinning of the flux distribution in the fluxoid lattice against Lorentz-like electromagnetic forces.

The Lorentz force density is[1]

$$F = \frac{\gamma}{c} \vec{J} \times \vec{B} \approx \frac{JB}{c} \quad , \tag{1}$$

where the effect of the equilibrium magnetization is included by introduction of the factor $\gamma = [\partial B(H)/\partial H]_{eq}$. In sufficiently high fields $\gamma \approx 1$. The approximate scalar form on the right of Eq. (1) holds for J and B mutually perpendicular. The critical force density F_c or corresponding critical force density $J_c \approx cF_c/B$ is loosely defined as the value of F(B) or J(B) at which dissipation becomes significant so that persistent currents decay at a significant rate, or a voltage appears along a wire carrying a transport current. Since Maxwell's equations give $J = (c/4\pi)\, \nabla \times \vec{B}$, or in simplified scalar form, $J = (c/4\pi)\, dB/dx$, magnetic cycling of a hard superconductor should produce a distribution of induced currents of magnitude up to J_c with corresponding nonuniform magnetic field distribution. The condition in which the current density is everywhere equal to the critical current density is designated as the "critical state" although it is not a uniform equilibrium state in the thermodynamic sense.

The magnitudes of the critical current density can be determined either by analysis of hysteretic magnetization curves produced by magnetic cycling or by the measurement of the transport current densities at which losses become appreciable in ribbons exposed to magnetic fields.

This explanation of critical currents was advanced by Bean[2] and by London,[3] and developed by Kim, Hempstead, and Strnad,[4] and has been amply substantiated by many

1. J. Friedel, P.G. deGennes, and J. Matricon, Appl. Phys. Letters 2, 119 (1963).
2. C.P. Bean, Phys. Rev. Letters 8, 250 (1962).
3. H. London, Phys. Letters 6, 162 (1963).
4. Y.B. Kim, C.F. Hempstead, and A.R. Strnad, Phys. Rev. Letters 9, 306 (1962).

subsequent experiments. Of interest amongst the earlier experiments are direct observations of the predicted field gradient by Coffey,[5] and a direct comparison of the critical transport current density with the current density deduced from magnetization measurements by Fietz, Beasley, Silcox and Webb,[6] in addition to various experiments by Kim and co-workers.[7] At least for large values of the Ginzburg-Landau parameter, κ, and internal fields well above the lower critical field but with $B < H_{c2}$, critical state concepts appear to describe adequately hard superconductors. However, in low κ materials and very thin filaments, hysteretic surface currents make significant contributions if the bulk J_c is small.

At temperatures above absolute zero Anderson[8] predicted that thermal activation assisted by the Lorentz force would lead to creep of flux in the neighborhood of the critical state. The general ideas of a thermally activated process suggest a flux creep rate

$$R = R_o \exp^{-U/kT} \quad , \tag{2}$$

where R_o is a very large pre-exponential and U is an activation energy which in this case is approximately

$$U = U_p - JB\gamma VX/c \quad , \tag{3}$$

where U is composed of an interaction energy between the pinning inhomogeneities and the fluxoid lattice, U_p, reduced by the Lorentz force per unit volume $JB\gamma/c$ times an activation volume V and a pinning length X. It turns out that $U \gg U_p$ and $U_p \approx JB\gamma VX/c$, and the rate R is appreciable only if $J > J_c$, so the critical state is reasonably well defined by putting $J = J_c$ such that

$$F_c = J_c B\gamma/c = \frac{U_p - kT \ln R_o/R_c}{VX} \quad , \tag{4}$$

where $R_c \ll R_o$ is a creep detection limit characteristic of experiments used to determine the critical current. Beasley, Labusch and Webb[9] have found that $U_p \gg kT \ln R_o/R_c$ so that the temperature dependence of F_c is dominated by the temperature dependence of various material parameters rather than by the explicit linear term in T.

II. CURRENT STATUS

In the present state of knowledge about critical currents in hard superconductors, the critical state concepts are well established and a great deal of practical data on critical currents in many hard superconductors have been obtained. The inhomogeneities that give rise to pinning of the fluxoid lattice have been explored in some detail and

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5. H.T. Coffey, Cryogenics 7, 73 (1967).
6. W.A. Fietz, M.R. Beasley, J. Silcox, and W.W. Webb, Phys. Rev. 136, A335 (1964).
7. See, for example, Y.B. Kim, C.F. Hempstead, and A.R. Strnad, Phys. Rev. 129, 528 (1963).
8. P.W. Anderson, Phys. Rev. Letters 9, 309 (1963).
9. M.R. Beasley, Ph.D. Thesis, Cornell University, 1968 (available as Materials Science Center Report 921 or AEC Report NYO-3029-29).

at least qualitative information is available about them. Dr. Livingston has thoroughly reviewed this information in the preceding lecture[10] and earlier in a review article with Schadler.[11]

A development has also appeared at this Summer Study that refocuses attention on the critical current problem. In recent, practical, high field superconducting devices, performance has been limited by instabilities that cause flux jumps that often cause catastrophic heating and loss of superconductivity. However, as reported here by Brechna, Hart, Wipf, and Smith in various contributions, there has been substantial recent progress in controlling this problem through appropriate geometric design. Thus, there is now again good reason to try to develop higher pinning force materials to obtain higher critical current densities.

However, a serious problem remains in applying the well established critical state concepts to hard superconductors. This problem is the lack of a satisfactory basis to calculate the critical current density to be obtained from even a simple array of localized pinning points. Livingston has described in some detail the various types of effective pinning points and our knowledge of their properties. The fact that we cannot simply add the pinning forces of all of the pinning points in a material to obtain F_c is clear. Instead we must take into account cooperative action amongst adjacent pinning points and fluxoids. Anderson[8] recognized this problem in 1963 in his formulation of flux creep by incorporating a sort of activation volume which he called the flux bundle. However, little subsequent progress with this problem has appeared in the literature.

III. SOME RECENT RESEARCH

At Cornell, M.R. Beasley, W.A. Fietz, R. Labusch, and I have been trying to understand the problem of determining the quantitative connection between J_c, or F_c, and the properties of the array of pinning points. This connection is complicated by the fact that the pinning process involves the cooperative interaction between a quasi-random array of pinning points and a nominally regular array of fluxoids. We have carried out systematic measurements of flux creep and critical current densities in alloys over a wide range of T, B, and κ using crystal dislocations as the pinning entities. It has been possible to calculate the pinning properties of dislocations better than other pinning entities so these otherwise complex objects were chosen as the best pinning entities to study.[12-15] From this work we have been able to extract information about the pinning process that has guided us in developing a simple model that seems to make the appropriate connection between the critical current density and the pinning points. This research is nearly complete, but manuscripts describing it are still in preparation. I will present here a preliminary summary of the essential ideas of the pinning model and the results of some of our pinning experiments.

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10. J.D. Livingston, these Proceedings, p. 377.
11. J.D. Livingston and H.W. Schadler, Prog. Mat. Sci. 12, 183 (1964).
12. W.W. Webb, Phys. Rev. Letters 11, 191 (1963).
13. E.J. Cramer and C.L. Bauer, Phil. Mag. 15, 1189 (1967).
14. R. Labusch, Phys. Rev. 170, 470 (1968).
15. J.S. Willis, J.F. Schenck, and R.W. Shaw, Appl. Phys. Letters 10, 101 (1967).

IV. PINNING MODEL

If we were to assume that the pinning force density is given simply by a linear superposition of the contribution of all the effective pinning points we would have a pinning force density

$$F_p = f_p N_p \quad , \tag{5}$$

where N_p is an effective density of pinning points and f_p is the maximum pinning force for each interaction. Since flux creep introduces negligible changes in the current density, the pinning force density F_p is essentially equal to the critical force density so we may take $F_p = F_c$.

Assuming that the pinning points consist of individual fluxoid-dislocation interactions, that the fluxoids are relatively stiff, and that their range of interaction with the dislocations is a distance d less than the fluxoid spacing, the effective density of interactions N_p becomes just the density of interactions with individual pinning points N_1, which is just the product of the length of fluxoid per unit volume B/φ_o, the effective area per unit length d, and the density ρ of dislocations threading the area, that is:

$$N_1 \approx (B/\varphi_o)\rho d \quad . \tag{6}$$

Typical magnitudes for our cases give $N_1 \approx 10^{11} \times 10^{11} \times 10^{-6} \approx 10^{16}\ \mathrm{cm}^{-3}$. Figure 1 depicts the geometrical model leading to Eq. (6).

If the fluxoid lattice were perfectly rigid, no net pinning could accrue from interaction with a random array of pinning points because in this case the total energy of the system would be independent of the position of the fluxoid lattice with respect to the pinning points. This astonishing result disappears if the lattice is even slightly deformable and a net pinning force is found, but the strength of the resulting pinning depends strongly on the magnitude of the deformation of the lattice introduced by the pinning interaction. In fact the ratio δ/d of the distance δ that a fluxoid can be displaced by the pinning potential to the width of the pinning potential d can be interpreted as the fraction of the points of interaction between fluxoids and obstacles that can become effective in pinning against an applied (Lorentz) force. This result is equivalent to replacing d by δ in Eq. (6).

The maximum deformation, δ, due to a single pinning point, depends on the strength of the interaction force per pinning point, f_1, and the effective stiffness of the fluxoid lattice represented by an elastic constant C times the nominal fluxoid spacing $(\varphi_o/B)^{\frac{1}{2}}$. Thus we can obtain, following the elasticity calculation of Labusch[16]

$$\delta \approx f_1 \frac{a}{C} \left(\frac{B}{\varphi_o} \right)^{\frac{1}{2}} \quad , \tag{7}$$

where a is a constant of order unity. Using[12,17] $f_1 \approx 10^{-8}$ dyn, and $C/a \approx 10^{6}\ \mathrm{dyn \cdot cm^{-2}}$, we find the extremely small displacement $\delta \approx 10^{-9}$ cm. A similar calculation in the low field limit using the line energy or tension of a single fluxoid as the restoring force yields essentially the same kind of result. Thus the fluxoid lattice is very rigid and

16. R. Labusch, private communication.

17. R. Labusch, Phys. Stat. Sol. 19, 715 (1967); additional results for $H \sim H_{c2}$ to be published.

the pinning process must involve a statistical average over all of the interaction forces sampled by this relatively rigid net.

Using Eq. (7) for δ, an efficiency factor $W = \delta/d$ can be defined as follows:

$$W = (f_p/d)(a/C)(B/\varphi_o)^{\frac{1}{2}} \quad , \tag{8}$$

where $f_p = f_1$ is the maximum pinning force of an individual dislocation-fluxoid interaction.

Thus, the pinning force density given by Eq. (5) is simply reduced by the efficiency factor W giving

$$F_p = f_p N_p W \quad . \tag{9}$$

Assuming $f_p = f_1$, $N_p = N_1$ as given by Eq. (6), and W as given by Eq. (8), the effective pinning force density is

$$F_p = f_1^2 \, \rho (B/\varphi_o)^{3/2} (a/C) \quad . \tag{10}$$

Thus we find the remarkable result that the pinning force density is proportional to the square of the strength of individual pinning points because of the appearance of f_1 in the efficiency factor of Eq. (9).

However, Labusch has carried out a detailed statistical analysis of this many-body problem[16] indicating that to lowest order in the pinning point density there is no pinning, i.e., $F_p = 0$, unless the quantity that we call an efficiency factor W is greater than 1. In other words, the elementary calculation indicates that it is necessary that distortions of the fluxoid lattice be larger than the width of the pinning potential for pinning to occur, or $\delta > d$. However, in usual materials $f_1 \sim 10^{-8}$ dyn, and $d \sim 10^{-6}$, and we have $\delta \sim 10^{-8}$, so $W \sim 10^{-2}$. Nevertheless, consideration of the effect of fluctuations in the pinning point distribution leads to pinning by even a random array as a first order effect. Assuming that only groups of pinning points strong enough that $W \geqq 1$ are effective, we seek fluctuations that provide a net local excess of n favorable pinning points providing a net strength $f_p \rightarrow f_n \geqq n\, f_1$, where the number n is determined by the requirement that $W \geqq 1$. This configuration should occur with random probability $1/n^2$ so that the effective density of these pinning coefficients is $N_p \rightarrow N_n \geqq (B/\varphi_o)\rho d/n^2$. The criterion that $W \geqq 1$ yields

$$n \geqq \left[\frac{f_1}{d} \left(\frac{B}{\varphi_o} \right)^{\frac{1}{2}} \frac{a}{C} \right]^{-1} \quad . \tag{11}$$

The new values of N_p and f_p can be inserted in Eqs. (8) and (9) to obtain the pinning force density F_p. However, inserting $N_p = N_n$ and $f_p = f_n$ yields precisely the same pinning strength given by Eq. (11) since n cancels out exactly in the expression for F_p.

If the pinning points are not randomly arrayed, the pinning force is enhanced by the clustering since the number of favorable fluctuations then varies more like $1/n$ instead of $1/n^2$. In our measurements on severely cold-worked alloys we found $10^{-8} < f_1 < 10^{-7}$ dyn, while the calculated value is $f_1 \sim 10^{-8}$, suggesting that some dislocation ordering occurs as is expected.

The collective pinning represented by the pinning force $f_p = nf_1$ in a volume containing between n and n^2 pinning interactions seems to be simply an explicit description of a property of the "flux bundle" hypothesized by Anderson in his original theory of flux creep.[8] The size of this bundle can be estimated from our measurements on niobium-titanium alloys[18] using Eq. (6), which give $n \sim 10^2$. Thus, $n^2 = 10^4$, and the "bundle volume" $V_p \approx N_p^{-1} n^2 \approx n^2/[(B/\varphi_o)\rho d] \approx 10^{-12}$ cm^3. From measurements of flux creep in PbTl alloys subjected to severe plastic deformation at low temperatures to produce pinning structures similar to ours, Beasley, Labusch and Webb[19] observed volumes $\sim 10^4$ times the volume per pinning point and pinning energies $\sim 10^2$ times the energy per pinning point under comparable conditions. This comparison suggests that the cooperative pinning in our analysis of the critical pinning force really corresponds to the activation volume or "flux bundle" invoked by Anderson to understand flux creep.

V. RESULTS OF CRITICAL PINNING FORCE MEASUREMENTS

We have deduced critical pinning force densities from hysteretic magnetization curves on niobium alloys over a wide range of the Ginzburg-Landau parameter, κ, magnetic field, H, and temperature, T. A sample of this data is shown as a logarithmic critical current density plot in Fig. 2, and as a critical pinning force density plot in Fig. 3. The fields have been normalized by dividing by the temperature dependent upper critical field H_{c2}. Although the critical currents show the usual plateau on a logarithmic scale, the pinning force invariably peaks at $B/H_{c2} = 0.6$ in these materials. This and all other features of the results support a well-defined set of scaling laws in which the pinning force density is given, the separable product of factors depending on temperature, normalized field and κ. The temperature dependence is accurately represented in terms of the temperature dependence of H_{c2}. Thus we found

$$F_p \propto k(\kappa)\, H_{c2}^{5/2}\, g(B/H_{c2}) \quad , \tag{12}$$

where the function $g(B/H_{c2})$ has the shape shown in Fig. 3 and $k(\kappa)$ is something like $\kappa^{-\gamma}$ with $1 < \gamma < 3$. The factor $[H_{c2}(T)]^{5/2}$ is established with high precision, $g(B/H_{c2})$ is determined to better than ± 15% for $0.2 < B/H_{c2} < 0.9$, but $k(\kappa) = \kappa^{-\gamma}$ is only a qualitative form. The most remarkable result is the precise temperature dependence fit given by the factor $[H_{c2}(T)]^{5/2}$. This is illustrated by the logarithmic plots in Fig. 4 where straight lines with slope 5/2 are drawn through the data.

To compare these data with the pinning model previously described, it is necessary to work out the temperature and field dependence of the fluxoid lattice stiffness represented by the factor a/C, and of the pinning force, f_1, that appear in Labusch's[17] calculation of a/c and Webb's[12] second order elastic calculation of f_1. Combining the field, temperature, and κ dependence of all of the factors we find that the pinning model gives the proportionality

$$F_p \propto H_{c2}(T)^{5/2} \cdot \begin{cases} (B/H)/\kappa^2 & : \ B \text{ small} \\ 1/\kappa^2 & : \ B \sim H_{c2}/2 \\ (B/H_{c2})(1 - B/H_{c2})/\kappa^3 & : \ 1 - B/H_{c2} \ll 1 \end{cases} \tag{13}$$

in satisfactory agreement with the experiments.

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18. W.A. Fietz and W.W. Webb, submitted for publication.

19. M.R. Beasley, R. Labusch, and W.W. Webb, private communication.

VI. DISCUSSION AND SUMMARY

Clearly the empirical model for critical pinning force that has been suggested by Fietz and Webb,[18] on the basis of Labusch's theoretical investigations,[16] fits the wide range of data they collected. It also fits data obtained by Coffey on a magnet wire alloy[20] and seems to be consistent with a great deal of earlier data on materials with other types of pinning points except that the detailed shape of the field dependence of F_p given by the factor $g(B/H_{c2})$ depends on materials preparation.

For example, T. Hall (unpublished) annealed some of Fietz's niobium alloys at various temperatures and found that a new maximum of F_p developed at $B/H_{c2} \cong 0.4$ as the peak at $B/H_{c2} \cong 0.6$ decreased, until even the new peak became very weak as annealing was made more complete. During this change the factor $[H_{c2}(T)]^{5/2}$ represented the temperature dependence of the first peak rather well, although the exponent for the second peak may be as low as about two.

Although the temperature dependence of a/C should remain the same for most superconducting alloys, various pinning entities should give different temperature dependences for f_1. This is yet to be tested systematically.

In the case of the superconducting intermetallic compounds, such as V_3Si and Nb_3Sn, based on the beta-tungsten structure, the situation seems to be different. It has been suggested that grain boundaries or nonsuperconducting inclusions provide the observable pinning and that the critical current is limited by Silsbee's hypothesis in part of the measured range.[21] Brand and Webb found that in V_3Si pinning is also enhanced by the occurrence of the martensitic phase transformation to a tetragonal structure.[22] Some uncertainties about pinning in these compounds remain.

Special pinning properties may lead to "peak effects" in the pinning strength[10] with unique temperature dependence, and it may be useful to exploit them or to look to the sort of models described here for ways of decreasing the temperature dependence of F_p in order to reduce thermal instabilities, and for ways of increasing the critical current density. Composite superconductors may provide some of these properties.

ACKNOWLEDGMENTS

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20. H.T. Coffey, Phys. Rev. 166, 447 (1968).
21. See contribution by G.D. Cody, these Proceedings, p. 405.
22. R. Brand and W.W. Webb, to be published in Solid State Communications.

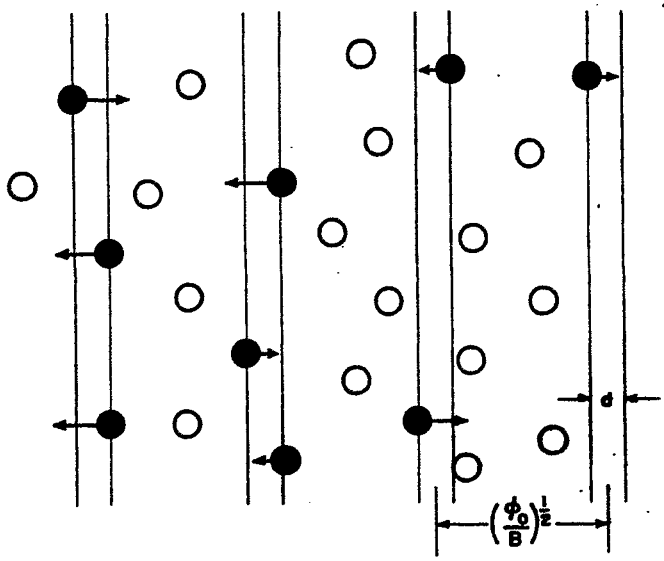

Fig. 1. Schematic diagram of the interaction of fluxoids with localized pinning points. The interaction range of pinning potential is represented by the effective width d of the fluxoids which are spaced a distance $(\varphi_0/B)^{\frac{1}{2}}$. The pinning points are supposed to be repulsive potential lines perpendicular to the paper. Thus the direction of the interaction force on the fluxoids is indicated by the arrows. Only the blackened pinning points are close enough to interact in this section. On the average in this rigid lattice model the net force on the fluxoid lattice is zero.

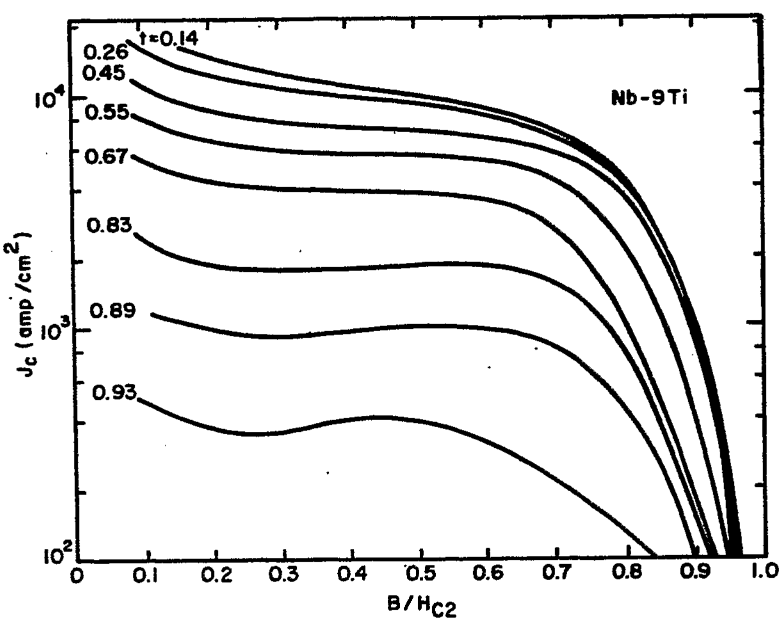

Fig. 2. Logarithmic plot of the critical current density as a function of normalized field B/H_{c2} in severely plastically deformed alloy of niobium with 9% titanium at various normalized temperatures $t = T/T_c$.

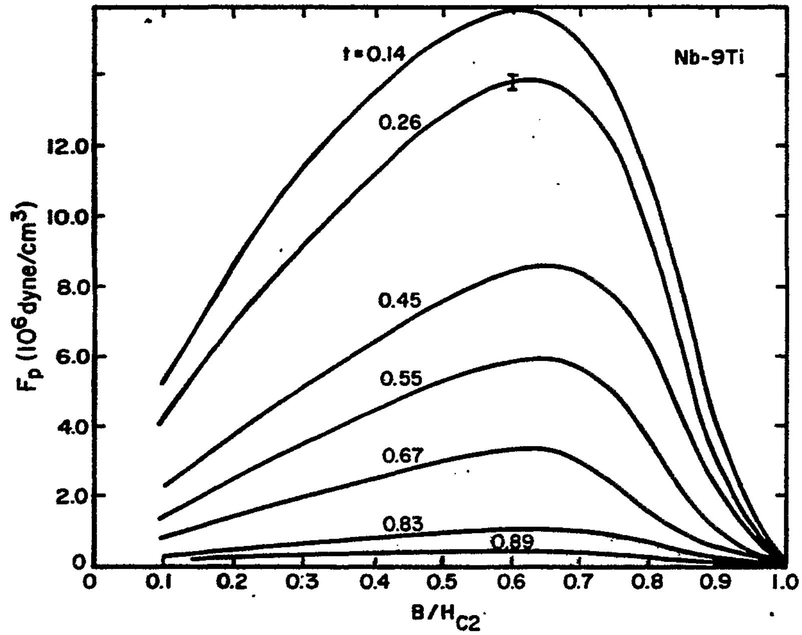

Fig. 3. Pinning force density as a function of normalized field B/H_{c2} in a severely plastically deformed alloy of niobium with 9% titanium at various normalized temperatures $t = T/T_c$. The normalizing value of H_{c2} is taken as $H_{c2}(t)$.

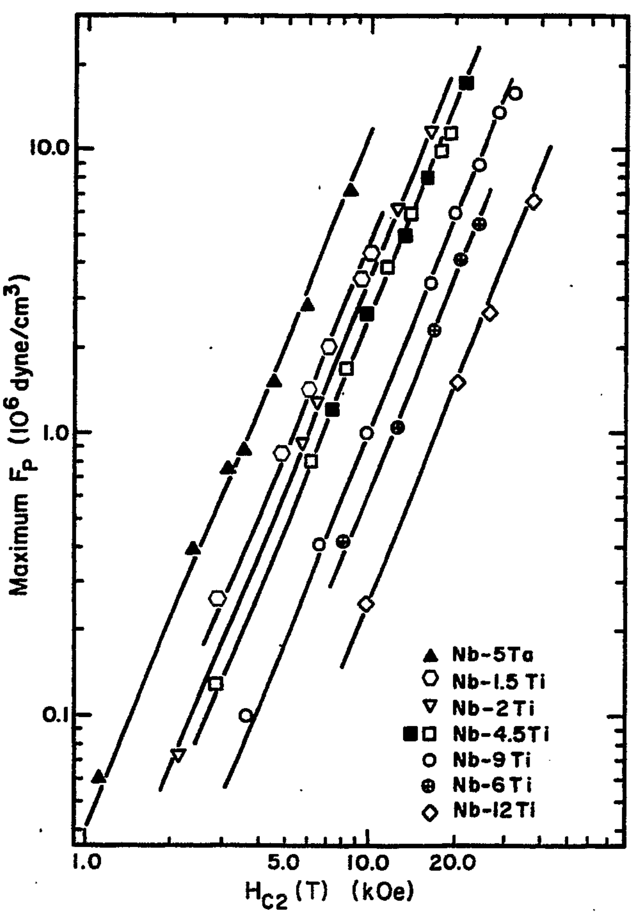

Fig. 4. Logarithmic plot of maximum value of the volume pinning force F_p(max) as a function of $H_{c2}(t)$. Lines have slope 5/2.