MAGNETIC INSTABILITIES AND SOLENOID PERFORMANCE: APPLICATIONS OF THE CRITICAL STATE MODEL

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I. INTRODUCTION

In this informal review I intend to discuss our understanding of the magnetic instabilities which occur in high field superconductors and the implications of our understanding for the design of high field, high current density solenoids. The approach will be mainly tutorial; the bibliography will allow reference to useful review articles and to some of the original sources.

After first mentioning very briefly the stability problems encountered in practical high field superconducting solenoids, we shall review the model which will form the basis of our discussion of magnetic instabilities: the critical state model. After illustrating the use of this model for isothermal situations, we shall consider the much more complicated situation in which the temperature is not constant. We shall find that under some conditions a small disturbance, say a small temperature rise, will grow catastrophically, driving the material into the normal state; the system will be unstable. We shall determine the conditions necessary for stability in certain limits, limits for which we can make simple calculations, limits yielding results useful for solenoid design.

The history and present state of development of solenoids has recently been reviewed very thoroughly by Chester.[1] This material will not be repeated here.

The central problem is the loss of performance of a conductor when it is wound into a solenoid, the degradation effect. As an example, a conductor formed of a high field superconductor may carry 40 A without a measurable voltage drop in a magnetic field of 40 kOe when tested in the form of a short length. The same conductor systematically will carry less than 20 A in a solenoid generating 40 kOe. This kind of performance degradation has been observed in each of the materials used in the construction of high field solenoids; it is a general phenomenon. It has been found experimentally that the superconducting-to-normal transitions leading to this limited performance occur in the lower field regions of the solenoid, not in the high field regions in which the critical current density of the superconductor is generally at its lowest.

The degradation effect can be alleviated to some extent by plating or coating the conductor with a pure normal metal such as copper or silver, or by winding into the solenoid layers of pure normal metal. Further, in some cases, a considerable increase in performance is obtained by running a solenoid in superfluid helium. Fine wire conductors apparently suffer less degradation than larger conductors.

The reader is referred to Chester's review[1] for details and references for the above observations. We shall now turn to a description of the critical state model, the model on which we plan to base our discussion of magnetic instabilities.

II. THE ISOTHERMAL CRITICAL STATE MODEL

Soon after the demonstration of the high field capabilities of Nb_3Sn and of alloys such as NbZr, it was found that a rather simple phenomenological model,[2] here called the critical state model, allows one to predict the magnetic behavior of a high field superconductor in terms of a simple empirical parameter. The basic assumption of the critical state model is that a changing flux density in a high field superconductor induces persistent currents up to a limiting or critical current density, J_c. These persistent currents are induced to flow according to Lenz's law in such a way as to minimize the change in the flux linking the sample. That portion of a superconductor carrying this limiting current density is said to be in the critical state. The parameter J_c depends on both temperature and magnetic field, though in some cases it is reasonable to assume that J_c is independent of field. Often J_c is assumed independent of field merely in order to simplify calculations.

Subsequently this phenomenological critical state model received a more physical basis. For an ideal, defect free, sample of a type II superconductor ($H_{c1} < H < H_{c2}$), the flux filaments threading the sample[3] are free to move about in the sample subject only to their mutual repulsive interactions[3,4] and to a viscous drag[5] which can be characterized by a flux flow resistivity, ρ_f. Nonuniform distributions of flux filaments, equivalent to field gradients or bulk currents, disappear quickly, with a time constant determined by ρ_f. However, for nonideal type II superconductors, persistent bulk currents or field gradients can occur when the flux filaments are pinned or trapped by defects in the structure of the material.[6] Such defects can be voids, normal inclusions, grain boundaries, compositional variations, dislocations, etc. A large enough bulk current density or field gradient can free the flux filament from the defect so that it can move; this critical bulk current density is J_c. The pinning and freeing of the flux filaments may well be a thermally activated process, a process of flux creep.[7] This process, and thus the critical current density itself, is temperature dependent. If the controlling process is flux creep, there is no true persistent critical current density; flux motion and current decay will continue indefinitely, albeit at a very slow and ever decreasing rate. In actual fact, for most purposes the currents can be considered as persistent below a given J_c. We shall return to this point in a later section.

We have up to this point discussed the critical state model in terms of flux motion or of induced persistent currents. It will be convenient throughout most of this review to think in terms of the relation between the electric field in the material and the current density. We can state the critical state model with some simplifications as follows: The electrical and magnetic properties of the superconductor are characterized by the nonlinear relation between the electric field and the current density indicated in Fig. 1. No current flows in the superconductor unless there has been a change in flux linkage. While the flux linkage is changing, the electric field and current density are related by the above relation and Maxwell's equations; note that local heating occurs, $P = E \cdot J$, even though we are dealing with a superconductor. When the steady state has been obtained, we find that a persistent current density $\pm J_c$ is flowing throughout those regions in which changes in flux linkage or magnetic field are occurred. The sign is given by the sign of the last nonzero electric field. No current is flowing where no changes in flux linkage occurred. The static magnetic properties are determined by Maxwell's equations and the distribution of current density: $0, \pm J_c$. If one restricts himself to isothermal field or current changes, he can easily calculate the final field and current distribution. We call this limit the isothermal critical state model.

In order to illustrate the application of the isothermal critical state model, let us work with the model in its simplest form: J_c independent of H, and ρ_f infinite. Thus the E-J relation is as shown in Fig. 2.

Let us consider a slab of a high field superconductor cooled in a field H_o parallel to the plane of the slab ($H_{c1} \ll H_o < H_{c2}$), Fig. 3a. Let us consider the isothermal response of this slab to changes in the applied field. Increasing the applied field causes flux penetration into the sample. For this geometry the Maxwell equation $\nabla \times \vec{H} = 4\pi \vec{J}/10$ (practical Gaussian units) reduces to $\partial_x H_z = -4\pi J_y/10$, yielding for the isothermal critical state model

$$\frac{\partial H_z}{\partial x} = 0, \mp \frac{4\pi}{10} J_c \quad ,$$

where the sign can be chosen by Lenz's law. A change in applied field ΔH_o thus yields a triangular penetration of field as indicated in Fig. 3b. The appropriate current distribution is also indicated. The depth of flux penetration, δ, is given by

$$\delta = \frac{\Delta H_o}{\frac{4\pi}{10} J_c} \quad .$$

For a sufficiently large change in applied field, the flux penetration from the two sides meet, and the field and current distributions are as indicated in Fig. 3c. Lowering the field from the peak field (Fig. 3d) causes flux to move out of the sample. Starting at the surface of the sample and moving inward, there is a reversal of the local current flow.

It is possible to calculate from this model the field distribution, the local flux penetration or flux linkage, and thus the local heating. For more detailed descriptions of the application of the model and of the results obtained see Bean,[2] London,[2] Kim et al.,[2] Hancox,[8] and Hart and Swartz.[9]

Let us now show similar current and field distributions for a cylindrical wire carrying a transport current. Let us consider the wire to have been cooled in a magnetic field H_o parallel to the axis ($H_{c1} \ll H_o < H_{c2}$). Let us apply a transport current parallel to the axis and plot the current distribution and circumferential field or self-field distribution. Before any current is applied there is no self-field (Fig. 4a). Upon the application of a current ($I < \pi R^2 J_c$), a flow of current density, magnitude J_c, penetrates from the surface (Fig. 4b) to a depth such that $J_c \cdot \pi(R^2 - R_1^2) = I$. The self-field is as indicated. The maximum current in the isothermal model is $\pi R^2 J_c$, when $R_1 = 0$. If, after having reached a peak current $I_o < \pi R^2 J_c$, the current is decreased, a current reversal penetrates from the outside as indicated in Fig. 4c. Again it is possible to calculate local field and current distributions, local flux penetration (including the instantaneous voltage drop down the wire), and local heating. It is possible to carry out the calculations for fields perpendicular to the axis as well. Pertinent references are Refs. 2, 8, and 9, as well as Grasmehr and Finzi.[10]

We have devoted this much space to the critical state model because it will be the basis of our following sections. We have to ask how well does it work in practice. Bean,[2] in static experiments at fields below 10 kOe, has found rather detailed agreement with this model, using J_c independent of H. Kim, Hempstead, and Strnad,[2] in static experiments covering a wider range of fields, found agreement with the critical state model predictions (with an important qualification) provided they used a field dependent J_c. In particular, they used a two-parameter relation: $J_c(H) = \alpha_c/(H + B_o)$. Other authors have found in different materials yet other field dependences for J_c. The field dependence of J_c is related to the defect structure yielding the flux pinning; this subject is discussed by J.D. Livingston in another paper in this Brookhaven Summer Study series. The important qualification in the work of Kim et al. is that under some conditions the isothermal critical state collapsed; flux rushed into the sample; a flux jump occurred. Thus we can say that for static experiments and isothermal conditions the isothermal critical state model is a satisfactory description. In this paper we shall try to determine the conditions under which the isothermal conditions cannot be maintained.

Several experiments indicate that the critical state model is useful as well in cases in which smaller, more rapidly varying fields are involved. Grasmehr and Finzi[10] have measured the instantaneous voltage drop along a wire carrying a 0.8 kHz alternating current. In addition they calculated the voltage waveform using the simple form of the critical state model (Fig. 2); the agreement between the model prediction and the experimental results is excellent. Instead of measuring the voltage directly and comparing their results visually, they could have Fourier-analyzed their predicted voltage drop and measured the harmonic content of the experimental voltage. The latter is what Bean[2,11] did for the output of a pickup coil containing samples exposed to alternating fields. Bean calculated for the higher harmonics that

$$\underset{(n>1)}{V_{n_{odd}}} = \frac{\gamma \cdot h_o^2\, f \cdot 5}{J_c(H_{dc},T)\cdot(n-2)(n+2)} \; ; \qquad V_{n_{even}} = 0 \quad ,$$

where n is the harmonic, f is the fundamental frequency, h_o is the alternating field amplitude, and γ is a constant involving the geometry of the sample and the pickup coil. In Ref. 11 the experimentally observed results (f = 5 kHz) are compared with the above predictions. The agreement is excellent through the seventh harmonic; deviations become apparent as one moves towards the seventeenth harmonic.

Agreement was obtained in these dynamic experiments even though the effects of a finite ρ_f (Fig. 1) were (quite properly) ignored. As we shall see later, there are some situations in which the finite ρ_f is important. At this point let us simply note that where $J < J_c$ no flux motion occurs, but where $J > J_c$ flux moves according to a flux diffusion equation

$$D \frac{\partial^2 B}{\partial x^2} = \frac{\partial B}{\partial t} \quad ,$$

where

$$D = \frac{10^9\, \rho_f}{4\pi} \quad .$$

Thus the time constant for flux redistribution for the slab geometry illustrated in Fig. 3b is roughly $\tau_m \sim \frac{1}{2}\delta^2/D$. Estimates of ρ_f for useful high field superconductors can be made on the basis of the work of Kim et al.[5]

The critical state model as we have introduced it can be expected to apply best for defect-loaded very high field superconductors such as NbZr, NbTi, or intermetallic compounds such as Nb_3Sn. If the applied fields are several kOe or more, we are correct in neglecting surface effects[12,13] and the reversible magnetization characteristic of type II superconductors.[3]

III. ADIABATIC CRITICAL STATE MODEL AND MAGNETIC INSTABILITIES

In Section II we followed the field profile in a slab of material as we increased the applied field isothermally. We predicted a triangular field penetration until the two triangles met. A similar flux penetration is predicted for a cylindrical rod. While in some studies such flux penetration is observed,[2] in many other studies, especially of superconductors having very large critical current densities, the flux penetration follows the predicted behavior over a limited field change and then a sudden and nearly complete flux penetration occurs.[14] Coffey[15] has probed the field profile in a divided rod of Nb60%Ti upon changing the applied (axial) field. He finds that the initial triangular field penetration ($H_a < 14$ kOe) is essentially as predicted by the isothermal critical state model. At higher fields there is a marked change in the mode of flux penetration; at some field the isothermal profile collapses and flux rushes in to bring the internal field up to the applied field. A further field increase leads to a new triangular shaped flux penetration profile until another catastrophic flux penetration occurs. These unstable flux penetrations occur rather regularly with a field spacing of between 4 and 8 kOe. These are the flux jumps or magnetic instabilities which we wish to understand; we wish to be able to predict their occurrence in terms of the properties of the high field superconductor.

Let us start out by recognizing the difficulty in maintaining isothermal conditions while changing the field. As pointed out by Swartz and Bean,[16] and by Wipf,[17] the magnetic flux can under some conditions diffuse through the useful high field superconductors much faster than can the heat generated by the moving flux; thus the superconductor can heat appreciably if a sudden field change occurs. The consequence of such a temperature rise can be seen qualitatively by looking at Fig. 3b while recalling that the critical current density of a high field superconductor generally decreases with increasing temperature.[18,19] As we saw in Section II, $\delta(T) = 10\Delta H_o/4\pi J_c(T)$. Thus a temperature rise leads to a further field penetration, which generates heat leading to a greater temperature rise and, in turn, to a yet greater flux penetration, etc. This thermal-magnetic feedback can under some conditions lead to a thermal runaway, a catastrophic flux jump.

As the field applied to a high field superconductor is increased, flux penetrates the sample. This flux penetration, even in the absence of flux jumps, is not smooth; it is a noisy[20] random process accompanied by local fluctuations of flux density about the critical state profile. Our concern is whether or not these fluctuations grow into gross flux jumps. The general problem is a very complicated one, for we are dealing with two coupled diffusion processes: the diffusion of flux and the diffusion of heat, the coupling arising from the generation of heat upon flux motion and the decrease in flux pinning or critical current density with an increase of temperature. Little progress has been made in this general problem. We shall consider only extreme limits for which simplifications occur.

Let us first consider the instability problem in the limit that no heat flows out of the region of flux penetration, a quasi-adiabatic limit. Let us consider a semi-infinite block cooled in a field H_o applied parallel to the face of the block ($H_{c1} \ll H_o \ll H_{c2}$). Let us isothermally increase the applied field by an amount ΔH_o. The field profile is now that characteristic of the bath temperature T_1. We can consider the stability of this profile against small disturbances by calculating the energies involved in going from this profile to others at higher temperatures. In order to simplify the computational details, let us assume that the critical current density is independent of field and that the total heat developed in any flux penetration is spread evenly throughout the region of flux penetration, i.e. the new temperature is uniform across this region. This last rather artificial assumption will later be relaxed. We consider the two uniform temperature critical state profiles illustrated in Fig. 5. We are interested in the amount of heat developed by the motion of flux in going from profile 1 to profile 2, Q_m, and in the amount of energy absorbed in heating the sample from T_1 to T_2, Q_T. These quantities are illustrated schematically in Fig. 6. In our quasi-adiabatic limit only those profiles are allowed for which $Q_m = Q_T$. If the excluded field ΔH_o is small (Fig. 7a), Q_m is always smaller than Q_T and only one solution is allowed, $T_2 = T_1$. The system is stable against small disturbances. For a sufficiently large excluded field ΔH_o (Fig. 7b), another higher temperature profile is allowed (and in our simple case is clearly accessible); the system is unstable, for a small disturbance will start the system towards the higher temperature state. If the initial slope of the Q_m vs T_2-T_1 curve is greater than the initial slope of the Q_T vs T_2-T_1 curve, instabilities are possible; the extent of the flux jump depends on the nonlinearities of the curves, in particular on the temperature dependences of the heat capacity and the critical current density. In order to determine the important physical parameters, let us calculate Q_m and Q_T as functions of T_2 and T_1. Let us assume the critical current density to be decreasing linearly with temperature[18]:

$$J_c(T) = J_c(T_1) \left(1 - \frac{T - T_1}{T_u - T_1} \right) ,$$

where $T_u \equiv T(J_c = 0)$, and the heat capacity to be proportional to the cube of the temperature[21]:

$$C(T) = C_1 \cdot (T/T_1)^3 \quad (J/^oK\ cm^3) \quad .$$

We calculate Q_m by calculating the total energy input to the system using the Poynting vector $\vec{P} = 10\ \vec{E} \times \vec{H}/4\pi$, and then subtracting that portion of this energy going into the increase in the energy of the magnetic field:

$$\frac{10^{-7}}{8\pi} \int_0^\infty [H_2^2(x) - H_1^2(x)]\, dx \quad .$$

The remainder is the heat generated. We first calculate the various quantities per unit surface area. Using the Poynting vector we find:

$$\Delta W_{12} = \int_{t_1}^{t_2} P dt = \frac{10^{-7}}{4\pi} \int_{t_1}^{t_2} \dot{\varphi}(x=0) \cdot (H_o + \Delta H_o)\, dt$$

$$= \frac{10^{-7}}{4\pi} (H_o + \Delta H_o)\ \Delta\varphi_{12}(x=0) \quad ,$$

where $\Delta\varphi_{12}(x=0)$ is the total flux crossing the surface in going from profile 1 to profile 2. From the triangular profile (Fig. 5) we find $\Delta\varphi_{12}(x=0) = \Delta H_o(\delta_2 - \delta_1)/2$. Therefore the total energy input is:

$$\Delta W_{12} = \frac{10^{-7}}{8\pi} (H_o + \Delta H_o) \Delta H_o \cdot (\delta_2 - \delta_1) \ ,$$

where

$$\delta_2 = \frac{\Delta H_o}{\frac{4\pi}{10} J_c(T_2)} \quad ,$$

and

$$\delta_1 = \frac{\Delta H_o}{\frac{4\pi}{10} J_c(T_1)} \quad .$$

The total change in field energy is:

$$\Delta E_{12} = \frac{10^{-7}}{8\pi}\left\{\int_0^{\delta_2} [H_2^2(x) - H_1^2(x)]dx\right\}$$

$$= \frac{10^{-7}}{8\pi}\left\{\int_0^{\delta_2} [H_o + \Delta H_o(1 - x/\delta_2)]^2 dx - \int_0^{\delta_1} [H_o + \Delta H_o(1 - x/\delta_1)]^2 dx - \int_{\delta_1}^{\delta_2} H_o^2\, dx\right\} ,$$

or

$$\Delta E_{12} = \frac{10^{-7}}{8\pi}\left(H_o + \frac{\Delta H_o}{3}\right)\cdot \Delta H_o \cdot (\delta_2 - \delta_1) .$$

Therefore the total heat generation (per unit surface area) is:

$$Q_m^I = \Delta W_{12} - \Delta E_{12} = \frac{10^{-7}}{8\pi}\cdot\frac{2}{3}\cdot(\delta_2 - \delta_1)\cdot(\Delta H_o)^2 ,$$

and the heat generation per unit volume of flux penetration is:

$$Q_m = \frac{Q_m^I}{\delta_2} = \frac{10^{-7}}{8\pi}\cdot\frac{2}{3}\cdot(\Delta H_o)^2\left(1 - \frac{\delta_1}{\delta_2}\right) = \frac{10^{-7}}{8\pi}\cdot\frac{2}{3}\cdot(\Delta H_o)^2\cdot\frac{T_2 - T_1}{T_u - T_1}$$

for $T_2 \leq T_u$. The energy absorbed in heating the sample, Q_T, is simply

$$\int_{T_1}^{T_2} C(T)dT .$$

For our assumed C(T) we obtain:

$$Q_T = C_1(T_2^4 - T_1^4)/4T_1^3 .$$

Let us now calculate the excluded field below which no instabilities occur, the excluded field for which the initial slopes of Q_m and Q_T are identical. We have:

$$\frac{10^{-7}}{8\pi}\cdot\frac{2}{3}\cdot\frac{(\Delta H_o)^2}{T_u - T_1} = C(T_1) = C_1$$

and thus:

$$\Delta H_o(\text{instability}) = \left\{12\pi\, C_1\cdot 10^7\cdot(T_u - T_1)\right\}^{\frac{1}{2}} .$$

Noting that

$$\frac{1}{T_u - T_1} = -\frac{1}{J_c}\frac{\partial J_c}{\partial T} ,$$

we can write a slightly more general result:

$$\Delta H_o(\text{instability}) = \left\{12\cdot\pi\cdot10^7\cdot C_1\cdot\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)^{-1}\right\}^{\frac{1}{2}} \quad .$$

For a given $C(T)/C_1$ the extent of a flux jump for a given starting ΔH_o and an isothermal profile depends on the ratio of ΔH_o to ΔH(instability) and on the ratio of T_u to T_1. For $C \propto T^3$ and for T_1/T_u much less than one, flux jumps for $\Delta H_o/\Delta H$(instability) just greater than one lead to insignificant flux penetration and heating; for T_1 near T_u any flux jump leads to complete flux penetration and to a final temperature above T_u.[16] Even at the lower temperature a sufficiently large ΔH_o can lead to a complete or full flux jump. In Fig. 8 we illustrate this dependence of the size of the flux jump on ΔH_o; here $T_u/T_1 = 5$. Wipf and Lubell[22] have pointed out that in order to have a full flux jump for the present geometry the following condition must be satisfied <u>in addition</u> to having $\Delta H_o/\Delta H(\text{instability}) \geq 1$:

$$10^{-7}\,\frac{(\Delta H_o)^2}{8\pi} \geq \int_{T_1}^{T_u} C(T)\;dT \quad .$$

Examining our result for ΔH(instability) we see three things: 1) the higher the heat capacity the higher the stable excluded magnetic field; 2) the smaller the decrease in J_c with increasing temperature, the higher the stable excluded field; in fact if $\partial J_c/\partial T \geq 0$ the system is inherently stable against small disturbances; and 3) for this semi-infinite geometry the actual magnitude of J_c cancels out.

Let us now remove the artificial assumption of a uniform final temperature; let us assume no heat flow at all, a locally adiabatic problem. Swartz and Bean[16] have solved this problem exactly. They derive and solve a second order differential equation for the flux crossing a surface $\Delta\varphi(x)$; the equation is of the form

$$\frac{\partial^2\Delta\varphi(x)}{\partial x^2} = -\frac{1}{\lambda^2}\,\Delta\varphi(x) + \text{const.} \;,$$

where λ is a characteristic distance for flux jumps:

$$\frac{1}{\lambda^2} = \frac{4\pi}{10^9 C}\cdot J_c^2\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right) \quad .$$

Their solution indicates that instabilities can occur only if the critical state penetration distance δ is greater than $\frac{1}{2}\pi\lambda$. This leads to a stable excluded field limit, similar to our ΔH(instability), which they call H_{fj}:

$$H_{fj} = \left\{\pi^3\cdot10^7\cdot C\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)^{-1}\right\}^{\frac{1}{2}} \quad .$$

This expression differs from our approximate one only by the factor $(\pi^2/12)^{\frac{1}{2}}$. Wipf and Lubell,[22] Hancox,[23] and Lange,[24] have derived very similar results. Swartz and Bean have considered in the same limit the size of a flux jump, the effects of different temperature dependences for the heat capacity, and the effect of having a slab of finite thickness. If the thickness of a slab is less than $\pi\lambda$, then no flux jumps can occur, for the flux penetration distance can never reach the critical value. Another way of saying the same thing is that if $H^* < H_{fj}$ the system is stable, where H^* is the field at which the flux penetrating from the two sides meets at the center. Setting the thickness of the slab, w, equal to $\pi\lambda$, we obtain the first of several stability restrictions on the critical current density and the sample size:

$$J_c^2\, w^2 \le \frac{\pi}{4} \cdot 10^9 \cdot C \left(- \frac{1}{J_c} \frac{\partial J_c}{\partial T}\right)^{-1} .$$

This result implies that stability may be more easily achieved for a finite sample cooled in a large field than for one cooled in a small field, for J_c usually decreases in increasing fields.

Experimental tests of our results are by no means quantitative; see Swartz and Bean.[16] Though many have observed flux jumps, few have measured the pertinent properties of the samples under study, few have tried to maintain the adiabatic conditions assumed. Neuringer and Shapira[25] have observed a series of incomplete flux jumps in cylinders of NbZr; the spacings of the jumps seem to be about the value predicted by the expression for H_{fj}. The present author has found in magnetization studies of a composite superconductor in limited contact with the liquid helium bath that the introduction of a high heat capacity element such as mercury doubled the spacing of the flux jumps, but the studies were not quantitative. One of the better measurements is that of Hancox[26] who studied a sample of porous Nb_3Sn, again in limited thermal contact with the helium bath. He not only observed flux jumps but also measured the heat capacity and critical current density of the sample. The agreement with the picture presented here was at least semi-quantitative, i.e., within a factor of two. He found, as had Goldsmidt and Corsan,[27] that allowing liquid helium into the porous Nb_3Sn increased by an order of magnitude the stable excluded field, presumably because of the very large heat capacity of liquid helium.

Livingston[28] showed that in a PbInSn alloy properly heat treated it is possible to obtain $dJ_c/dT > 0$ over a limited range of field and temperature. He further pointed out that such a material should be stable in that region. The present author and Livingston, in an experiment designed to be especially sensitive to flux jumps, have studied the flux jump behavior of this material.[29] Flux jumps were clearly observed in those regions in which $dJ_c/dT < 0$; flux jumps were absent where $dJ_c/dT > 0$.

One can make very similar calculations for nonzero transport currents. Swartz[9] has performed such calculations for isolated round wires and for flat sheets or films. Here we shall simply remark that under some conditions a wire or film will quench at a current well below the isothermal critical current; in fact the quench current can actually decrease as the isothermal critical current is increased. Swartz tested his predictions using deposited NbO films and found agreements somewhat more quantitative than those we mentioned earlier.

Hancox[30] has applied concepts similar to these to wires in the winding of a solenoid. He uses the quasi-adiabatic approximation with which we started. In addition, he approximates a single layer wound from round wire as a thin sheet, thickness D, for which the field is parallel to the surface; the field is large enough for complete flux penetration. He finds in a calculation analogous to a calculation of ΔH(instability) that a winding is stable up to its full critical current provided that

$$J_c^2\, D^2 \le \frac{3}{4\pi} \cdot 10^9 \cdot C \cdot \left(- \frac{1}{J_c} \frac{\partial J_c}{\partial T}\right)^{-1} .$$

Thus stability is improved if the heat capacity is increased, if $-\partial J_c/\partial T$ is decreased, if D is decreased (a finer wire), and if J_c is decreased, e.g., by placing a field on the winding before energizing the winding, as has been discussed by the RCA group.[31]

Hancox also makes a more detailed calculation introducing the temperature dependence of the heat capacity, a calculation in effect of the extent of a flux jump. He considers the field and current density distribution illustrated in Fig. 9. Balancing the energy dissipated in the flux penetration against the increase in thermal energy he finds the following expression for the current degradation expected in a solenoid:

$$\frac{\pi D^2}{6}\left(J_{c1}^2 - J_{c2}^2 + 6J_{c2}^2 \ln \frac{J_{c1}}{J_{c2}} \right) = 10^9 \cdot \int_{T_1}^{T_2} C(T)\, dT \quad .$$

Here $\pi D^2 J_{c1}/4$ is the short sample current for the wire and $\pi D^2 J_{c2}/4$ is the quench current of the solenoid. Here we are allowing minor or partial flux jumps to occur; small enough flux jumps do not quench the solenoid, they merely give small voltage pulses across the coil[1] and perhaps a noisy field in the solenoid. A sufficiently large flux jump will quench the solenoid. At the Summer Study Hancox presented informally a very complete phase diagram for a wire wound solenoid indicating the conditions under which quenching instabilities are to be expected.

Again the agreement with experience is at best semi-quantitative; the predictions appear optimistic. Iwasa and Williams[32] suggest on the basis of their experiments that we may be optimistic in our predictions because we have always assumed that the flux jump is one-dimensional; we have ignored any variations of the flux penetration in the y or z directions. The importance of their suggestion is not yet clear.

Recalling Hancox's first expression we find essentially the same pertinent parameters determining flux jump stability: the heat capacity of the winding, the loss of J_c with increasing temperature, the size of the conductor, and the magnitude of J_c. Let us note that Hancox's stability expression for a slab carrying a transport current is about four times as severe as Swartz and Bean's similar expression for zero transport current (it would be exactly four if the same thermal approximations had been used). This factor of four can easily be reconciled if one recognizes that the field profile for a slab carrying the full isothermal critical current is similar to the profile for one half a slab carrying no transport current but exposed to a field greater than H^*. The factor is four because the dimension is squared in our expressions.

The stabilization of a solenoid against flux jumps by the use of the heat capacity of the winding is called adiabatic stabilization or enthalpy stabilization. It appears possible to make some conductors fine enough to make them stable[1]; the problem is that these fine conductors are embedded in parallel in a normal metal. In order to obtain the full advantage of the fine conductors there must be an effective transposition of the superconducting wires. Twisting is effective as far as uniform applied fields are concerned, but is not effective for transport currents. The problem is that without transposition there is an effective diamagnetism for times long compared to the period of field increase; this diamagnetism is temperature dependent and can still lead to instabilities. The effectiveness of twisting instead of the much more difficult transposition is yet to be determined.

IV. MAGNETIC INSTABILITIES: ATTEMPTS TO INCLUDE THE FINITE THERMAL AND MAGNETIC DIFFUSIVITIES

In the preceding section we have considered magnetic instabilities in the limiting case of the adiabatic critical state model. Though, as pointed out by Swartz and Bean, this is often a rather good approximation for Nb_3Sn and similar materials, there are some situations in which the nonzero thermal conductivity and the finite flow resistivity are of importance. The treatment of this problem is sufficiently difficult that little progress has been made. The most detailed and general attempt thus far is the work of Wipf.[17] Wipf's discussion is sufficiently complex that I shall refer you to his paper for details. (His introduction is a very good discussion of flux instabilities in terms of the motion and pinning of fluxoids.) Wipf analyzes the problem of a semi-infinite block excluding a magnetic field, assuming the heat capacity to be independent of temperature. Within this approximation he distinguishes three regions of excluded field:

1) $\Delta H_o < \Delta H$(full stability): No instabilities can occur.
2) ΔH(full stability) $\leq \Delta H_o < \Delta H$(limited instability): A flux disturbance can take place but the heat conduction out of the flux penetration region restores J_c before appreciable flux penetration occurs.
3) ΔH(limited instability) $< \Delta H_o$: If a flux disturbance occurs, the growth of the flux penetration region is too rapid to allow thermal recovery; a near adiabatic runaway instability or full flux jump can occur. (As stated above, this discussion neglects the increase in heat capacity with temperature.)

No simple expressions are given for the boundaries of these regions, except for ΔH(full stability), for which essentially our ΔH(instability) or H_{fj} is obtained. Speaking loosely, if the time constant for flux penetration into the conductor is much longer than the time constant for the diffusion of heat out of the conductor, ΔH(limited instability) can be appreciably greater than ΔH(full stability). The time constant for heat flow out of the superconductor depends in a rather simple way on the thermal conductivity and the geometry. The time constant for flux penetration depends on the geometry and on the effective flux flow or flux creep resistivity. Looking back at Fig. 1 we see that the flux motion time constant depends on ρ_f. For the useful high field superconductors, the thermal conductivities and flow resistivities are such that the thermal time constants are long compared to the magnetic time constants as long as the superconductor experiences an appreciable flux motion or electric field. In this case, i.e., for rapidly varying fields, the adiabatic limit is appropriate and ΔH(limited instability) is very close to ΔH(full stability). However, for very slowly varying fields, i.e., very small electric fields, flux creep may be the controlling mechanism; our Fig. 1 is then an oversimplification, there is no sharp corner in the E-J relation near J_c. In fact, both the flux creep model[7] and the present somewhat limited experimental evidence[7,33] indicate that for small electric fields we should write $\rho_E = \gamma E$. Thus for a slow enough rate of change of field the effective resistivity is small enough that the magnetic time constant becomes long compared with the thermal time constant; ΔH(limited instability) becomes appreciably greater than ΔH(full stability). This trend is often observed experimentally.

Of more practical interest, the effective magnetic time constant can be greatly increased by introducing pure normal metals in close magnetic coupling with the superconductor, increasing ΔH(limited instability) well above ΔH(full stability).

Let us now consider magnetic instabilities in the <u>extreme</u> limit that the magnetic time constant, τ_m, is much longer than the thermal time constant, an extreme limit yielding closed form answers of practical importance for certain structures. (It is a limit of interest to the present author.) Let us consider a slab of a high field superconductor of thickness or width w exposed to a magnetic field parallel to the plane of the slab, a geometry similar to that of Fig. 3. The superconducting or critical state properties of this material are described by Fig. 1 with J_c assumed independent of H. The thermal conductivity of the material is denoted by K (W cm^{-1} $^oK^{-1}$). There is in addition an interface barrier to the flow of heat out of the superconductor described by the linear relation: $\dot{Q} = h \cdot \Delta T_s$ (W cm^{-2}). Here ΔT_s is the difference between the surface temperature of the superconductor and the temperature of the surrounding medium, perhaps a liquid helium bath, perhaps a normal metal bonded to the superconductor. Initially let us assume that the flux penetration fronts from the two sides have reached a common point, as in Fig. 3c; the critical current density has been induced to flow throughout the entire sample. We shall neglect entirely any stability resulting from the heat capacity of the superconductor.

We shall apply a sudden small disturbance to the system and calculate whether or not a thermal runaway occurs. For convenience let the initial disturbance be a sudden small temperature rise ΔT. Our criterion for stability will now be a dynamic one, in contrast to the static criterion of Section III. The system will be considered dynamically stable if the temperature rise immediately decays back towards the original equilibrium temperature; it will be considered unstable if the temperature increases further yet. We shall neglect any nonlinearities in the problem. This is a rather conservative or pessimistic criterion.

In the extreme limit of a very long magnetic time constant (relative to the thermal time constants) the calculation is greatly simplified because we can then assume that the initial magnetic field profile and the initial current distribution determine the flux motion and heating rate for some time after the initial temperature rise. Upon a sudden increase in temperature there is a sudden decrease in the flux pinning in the superconductor, a sudden decrease in the critical current density, but not in the total current density; flux begins to move through the sample generating heat. Referring to Fig. 10, assuming that ρ_f is relatively unaffected by the change in temperature, we find that the initial heating rate is:

$$\dot{q}(x) = J\cdot E(x) = J_{c1}\cdot\rho_f\cdot(J_{c1} - J_{c2}) = J_{c1}^2\,\rho_f\cdot\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)\cdot\Delta T(x) \quad .$$

We see that the local heating rate is proportional to the local temperature rise. The dynamic stability of the system is determined by the relative magnitudes of the rates of heat generation and of heat removal. The barriers to heat removal are the internal thermal resistance of the material and the surface interface barrier. We shall solve the problem for the two limiting cases: 1) surface interface barrier predominant, and 2) the internal thermal resistance predominant. We shall quote the result for the intermediate case.

For the first limiting case let us assume that the effective barrier to heat removal is the interface barrier, $\Delta T = \dot{Q}/h$. We thus assume that the thermal conductivity of the plate is large enough that we have a uniform temperature across the plate. The stability criterion is obtained by writing an energy balance equation:

$$w\cdot C\cdot\frac{d(\Delta T)}{dt} = wJ_{c1}^2\rho_f\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)\cdot\Delta T - 2h\Delta T \quad .$$

The solution of this differential equation is an exponential:

$$\Delta T(t) = \Delta T_o\,\exp(\alpha t)\ ; \qquad t \ll \tau_m \quad .$$

We thus have dynamic stability if $\alpha < 0$:

$$\frac{wJ_{c1}^2\rho_f\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right) - 2h}{w\cdot C} < 0$$

or if:

$$J_c^2\,w < \frac{2h}{\rho_f\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)} \quad .$$

The result just found applies for arbitrary transport current for full flux penetration, $H > H^*$. If the flux penetration is incomplete, the width w is replaced by $2\overline{\Delta H}/(4\pi J_c/10)$ and the result is a limiting excluded field:

$$\overline{\Delta H}_s \leq \frac{4\pi h}{10 J_c \rho_f \left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)} \quad .$$

Here $\overline{\Delta H}$ is the average of the excluded fields on the two sides of the slab.

For our second case let us assume that the interface barrier is very small (h is large) and that the barrier to heat removal is determined by the thermal conductivity K. Again let us assume complete flux penetration, $H > H^*$, an arbitrary transport current, and a field independent J_c. We must now solve the thermal diffusion equation[34] with a heat generation term:

$$K \frac{\partial^2 \Delta T}{\partial x^2} - C \frac{\partial \Delta T}{\partial t} + \dot{q} = 0 \quad ,$$

where

$$\dot{q} = a\Delta T = J_{c1}^2 \rho_f \left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right) \Delta T \quad .$$

Our boundary conditions are that the surface temperatures ($x = \pm w/2$) are equal to the bath temperature and that the initial temperature, just after the heat pulse, is equal to ΔT_o. Our trial solution will be a time-dependent Fourier series. Choosing the origin of the coordinate system in the center of the plate, we can expand the initial temperature disturbance as a cosine series. Our trial solution will thus be the series

$$\Delta T(x,t) = \sum_{n=0}^{\infty} b_n e^{\alpha_n t} \cos\{(2n+1)\pi x/w\} \; ; \quad t \ll \tau_m \quad .$$

Introducing the general term into the differential equation we find that we have a solution provided that

$$\alpha = \frac{a - K(2n+1)^2 \left(\frac{\pi^2}{w^2}\right)}{C} \quad .$$

Our requirement for stability is that all terms in the sum decay; none blow up. The most severe requirement is the term n = 0. Thus the criterion for stability is that:

$$a - K\frac{\pi^2}{w^2} < 0 \quad ,$$

or

$$J_c^2 w^2 < \frac{\pi^2 K}{\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)\rho_f} \quad .$$

For incomplete flux penetration and zero transport current we again find a limiting excluded field:

$$\Delta H_s < \left\{ \frac{\pi^4}{25} \cdot \frac{K}{\rho_f} \cdot \frac{1}{\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)} \right\}^{\frac{1}{2}} \quad .$$

It is possible to solve our problem for arbitrary values of K and h, as long as the magnetic time constant remains long compared to the thermal time constant. The result for $H > H^*$ is an inequality:

$$\frac{(Ka)^{\frac{1}{2}}}{h} \cdot \tan \left[\frac{w}{2} \left(\frac{a}{K} \right)^{\frac{1}{2}} \right] < 1 \quad .$$

Again, for partial penetration and zero transport current, another inequality in which the stable excluded field is determined is:

$$\Delta H_s < \frac{4\pi J_c}{10} \cdot \left(\frac{K}{a} \right)^{\frac{1}{2}} \cdot \arctan \left(\frac{h}{(Ka)^{\frac{1}{2}}} \right) \quad ,$$

where

$$a \equiv J_{c1}^2 \rho_f \left(-\frac{1}{J_c} \frac{\partial J_c}{\partial T} \right) \quad .$$

We have used as our disturbance a uniform heat pulse. The same result would be obtained for a nonuniform heat pulse or for a uniform heating rate. For the latter see Carslaw and Jaeger,[34] p. 404. I must caution you that a disturbance in the form of an instantaneous field rise would not lead to this simple analysis, and might lead to slightly different numerical factors in the resulting expressions. The results we have obtained are pessimistic in that we have neglected any stabilizing influence of the heat capacity of the material.

Before applying these results to a practical system let us note the properties influencing dynamic stability. We see once again that a material having $\partial J_c/\partial T > 0$ is stable against small disturbances. We see once again that systems presenting small cross sections (small w) to the magnetic field tend to be more stable. The new results are that the larger the thermal conductivity, the smaller the boundary resistance, and the lower the effective resistivity, the more stable the system. In composite materials the inclusion of pure normal materials can greatly lower the resistivity and can sometimes increase the pertinent thermal conductivity.

Let us now attempt a simplified application of these dynamic stability concepts to instabilities in structures wound from wide, thin (e.g., ½ in. x 0.005 in.) composite tapes. Such tapes are available commercially and are in use at several laboratories, including AEC laboratories. In those portions of the magnet for which the field is parallel to the plane of the tape, the material is stable by adiabatic stabilization, for

$$J_s^2 \, d_s^2 < \frac{3 \times 10^9}{4\pi} \cdot C \cdot \left(-\frac{1}{J_c} \frac{\partial J_c}{\partial T} \right)^{-1} \quad ,$$

where d_s is the thickness of the superconductor, and J_s is the critical current density of the superconductor. However, for those portions of the magnet where there is an appreciable component of the field perpendicular to the plane of the tape the winding is not adiabatically stable. As described by Graham and Hart,[35] a winding formed of thin wide tapes is effectively a diamagnetic body for the component of field perpendicular to the tapes; its properties can be described by a critical state model in which the properties are averaged over the cross section. It is thus very similar to the slabs we have just been considering; the thickness of the slab, w, is the width of the tape. (See Fig. 11.) This composite diamagnetic body is both theoretically and experimentally adiabatically unstable (see Fig. 12); we must depend on dynamic stabilization, the transfer of heat ultimately to helium, to avoid flux jumps (see Fig. 13).

In a simple solenoid the less stable regions occur at the ends of the solenoid where the radial field component (perpendicular to the plane of the tape, parallel to the face of our equivalent slab) is the largest. In quadrupoles the largest perpendicular field can be at the most critical point in the winding, the region of highest winding current density. We shall not attempt an analysis of a real structure; we shall simply discuss the dynamic stability of a slab formed by stacking the tapes. There are several ways in which instabilities can occur; we shall take them in turn.

First, any heat generated because of a temperature rise must be removed from the superconductor itself to the neighboring normal metal. There are two barriers to the removal of this heat, a bond resistance and, more important, the very poor thermal conductivity of the superconductor itself. The following calculation applies to those regions into which the perpendicular (to the plane of the tape) field has penetrated. Assuming perfect magnetic coupling between the superconducting and the normal layers of the tapes, we can write for the local rate of heat generation in the superconducting material (see Fig. 11):

$$\dot{q}_s(y) = J\cdot E = J_s \left\{ \frac{J_s d_s \left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)\Delta T}{\frac{d_s}{\rho_s}+\frac{d_n}{\rho_n}} \right\} \equiv a\Delta T \quad .$$

Usually we can assume

$$\frac{d_n}{\rho_n} \gg \frac{d_s}{\rho_s} \quad .$$

Applying the same dynamic stability criterion as we did in our earlier calculations, we find the following restrictions on the properties of the superconducting layer in the tape:

1) $$J_s^2\, d_s^2 < 2h\,\frac{d_n}{\rho_n}\,\frac{1}{\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)} \quad ,$$

2) $$J_s^2\, d_s^3 < \pi^2 K_s\,\frac{d_n}{\rho_n}\,\frac{1}{\left(-\frac{1}{J_c}\frac{\partial J_c}{\partial T}\right)} \quad .$$

As I have mentioned before, these expressions are somewhat pessimistic. Chester[1] has presented an expression essentially the same as the second of these; his expression is more pessimistic than mine by a factor of $\pi^2/4$. This second equation, involving the very poor thermal conductivity of the superconductor, is very important because some of the commercial high current Nb_3Sn composite tapes presently used are just barely stable; some others are almost certainly unstable. Notice that for all other properties held fixed, the thicker the superconductor the greater the chance of instability. Notice also that the short sample or isothermal critical current of the tape is proportional to the product of J_s and d_s. Thus for constant width of tape there will be a tendency for tapes of higher critical currents to be less stable. The tape user cannot ask the manufacturer to deliver ever higher critical current tapes (of constant width) without running into instabilities at some point. These statements are not meant to stop all progress; they are just a warning. (Again, if $\partial J_c/\partial T > 0$, the system is stable.) We save our discussion of the factor d_n/ρ_n until later.

Now, assuming we have a <u>tape</u> which passes the above stability requirements, we can consider the dynamic stability of our composite slab. If we have a typical disk winding, one in which useful helium does not exist between the tapes, we must have a sufficient transverse thermal conductivity to assure that the heat reaches the tape edge

$$H_\perp < \left\{ \frac{\pi^4}{25} \frac{\bar{K}}{\bar{\rho}_f} \left(- \frac{1}{J_c} \frac{\partial J_c}{\partial T} \right)^{-1} \right\}^{\frac{1}{2}} ,$$

and a sufficient heat transfer to the helium bath to assure stability

$$H_\perp < \frac{4\pi\bar{h}}{10\bar{J}_c\bar{\rho}_f} \left(- \frac{1}{J_c} \frac{\partial J_c}{\partial T} \right)^{-1} .$$

Again, these requirements are not met trivially.

If one takes the care necessary to insure useful liquid helium between the tapes, he greatly increases the interface area and also removes the need for a good transverse thermal conductivity. The use of superfluid helium immediately suggests itself.[36] One still must get the heat out of the superconductor and across any barrier such as stainless-steel strengtheners, insulation, etc., into the liquid helium, as we calculated above.

In <u>each</u> of these expressions there appears a factor such as d_n/ρ_n, or $1/\bar{\rho}_n$. The magnet builder can influence this factor in two very useful ways. For those regions having large field components perpendicular to the tape he can either order his tape with enough pure enough normal metal bonded to the superconductor to insure stability, or he can co-wind into his disk, along with the composite conductor, a tape of pure normal metal. This extra tape need not be electrically or thermally bonded to the composite superconductor in order to influence d_n/ρ_n or $1/\bar{\rho}_n$. In our laboratory both techniques have been found quite successful; at this time, however, the amount of normal metal needed must be found empirically.

V. CONCLUSION

In considering magnetic instabilities and solenoids we have concentrated on understanding two extreme limiting cases, both of which are important in practice. In both limits we find that stable performance would be assured if we could find useful materials for which $\partial J_c/\partial T \geq 0$; I have hopes that such materials will someday be found.

In one limit, the adiabatic limit, we found that small enough conductors ($\lesssim 0.002$ cm diam) can be stabilized against flux jumps with their own heat capacity. There is a major qualification: if the fine conductors are connected electrically at more than one point, some form of transposition will be required to gain the full advantage of the small dimensions. There is some promise that such conductors will be forthcoming (see P.F. Smith's contribution to this Summer Study).

In the other limit, in which the winding is well cooled and the flux motion is slowed by a normal metal, we find that we can attain a useful dynamic stability[1] by the proper use of a very pure normal metal in close magnetic coupling with the superconductor. Again, fine dimensions offer advantages.[1] The positive permeation of a winding with liquid helium, particularly superfluid helium, should aid the attainment of stability.

R. Hancox has prepared, but not yet published, an excellent summary of the effects of instabilities on the performance of wire wound solenoids. P.F. Chester's review of superconducting solenoids[1] deserves careful reading for it has as one of its main themes the importance of magnetic stability for solenoids. Finally, S.L. Wipf's concise review of instabilities printed elsewhere in these Proceedings is recommended to the reader.

I would like to thank my many colleagues in the General Electric superconductivity effort for helping, and forcing me to learn about magnetic instabilities in high field superconductors.

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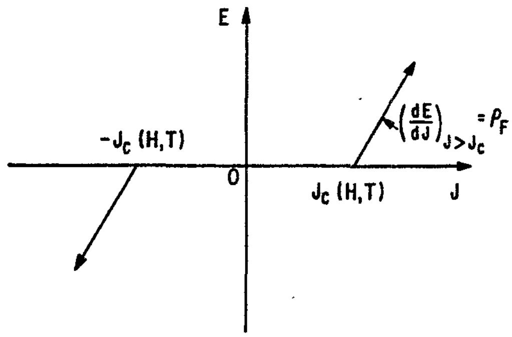


Fig. 1. The nonlinear relation between the electric field E (V/cm) and the current density J (A/cm^2). Here ρ_f (Ω·cm) represents the effective flux flow resistivity for the superconductor; in some cases ρ_f includes the effects of normal metals, i.e., in composite structures. This E-J relation may be considered one formulation of the critical state model.

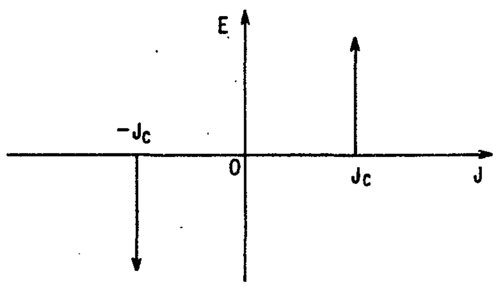


Fig. 2. A simplified relation between E and J which is often used in critical state model discussions. In this version the response of a superconductor to a field change is instantaneous.

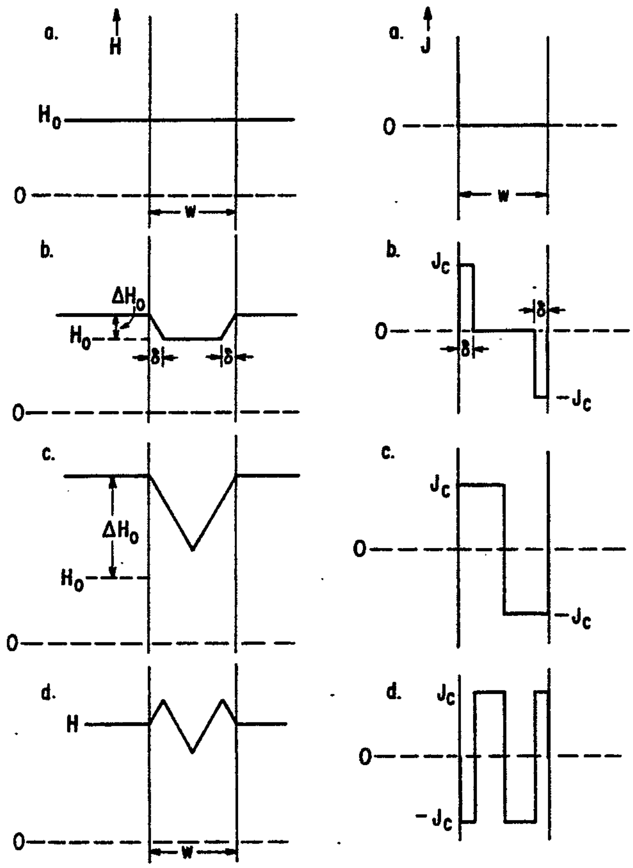


Fig. 3. The history dependent distributions of field and current in a high field superconducting slab with a magnetic field applied parallel to the plane of the slab. The detailed history is described in the text. The excluded field at which the flux fronts from the two sides meet ($\delta = w/2$) is called H^*. Thus in Fig. 3c, $\Delta H_o > H^*$.

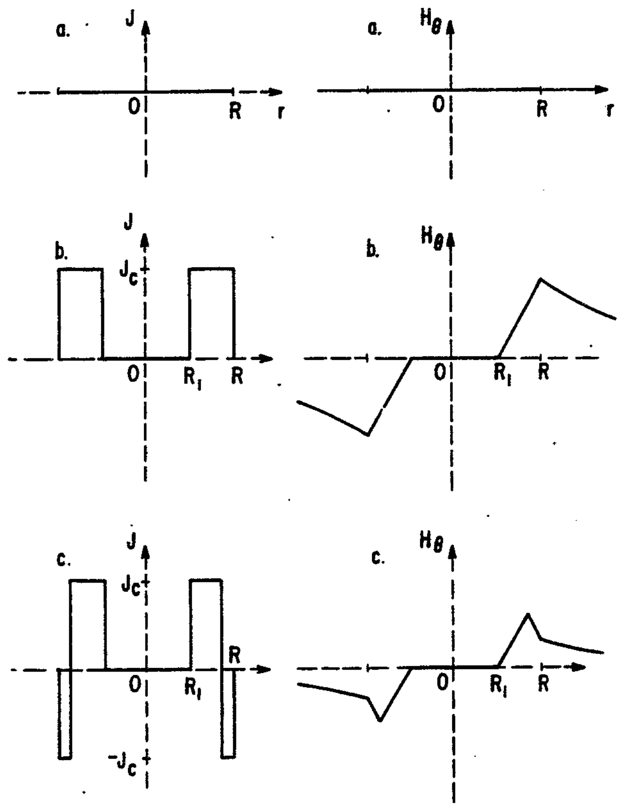


Fig. 4. The history dependent distributions of field and current for a round wire carrying a current. See the text for details.

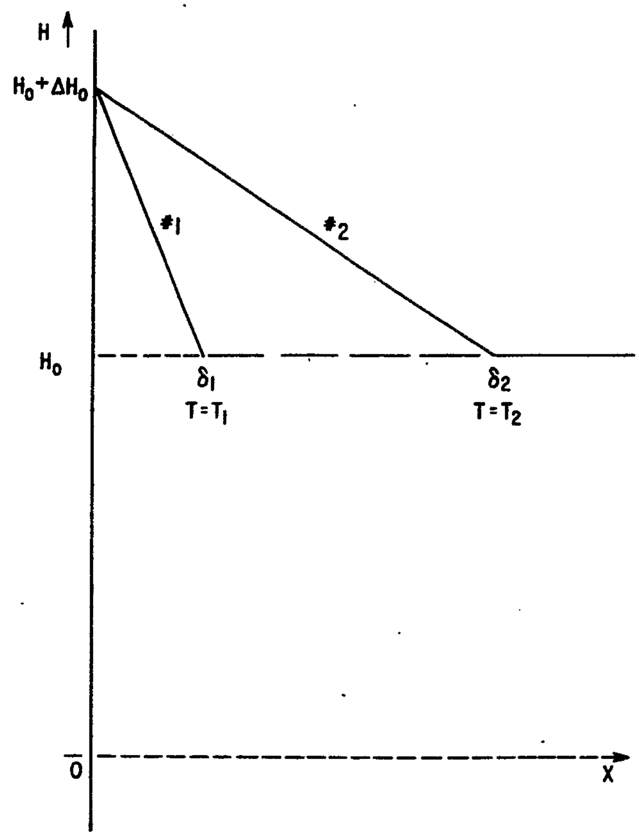
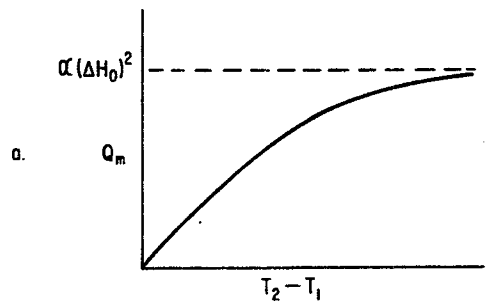


Fig. 5. Uniform temperature flux penetration profiles for a semi-infinite slab for two different temperatures.



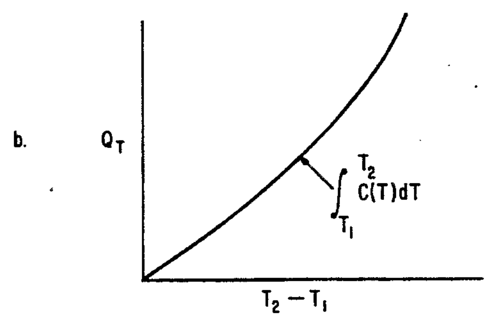


Fig. 6. Schematic illustrations of the heat generated, Q_m, and absorbed, Q_T, in going from one field profile, temperature T_1, to a new field profile, temperature T_2. The units of Q in this calculation are J/cm^3.

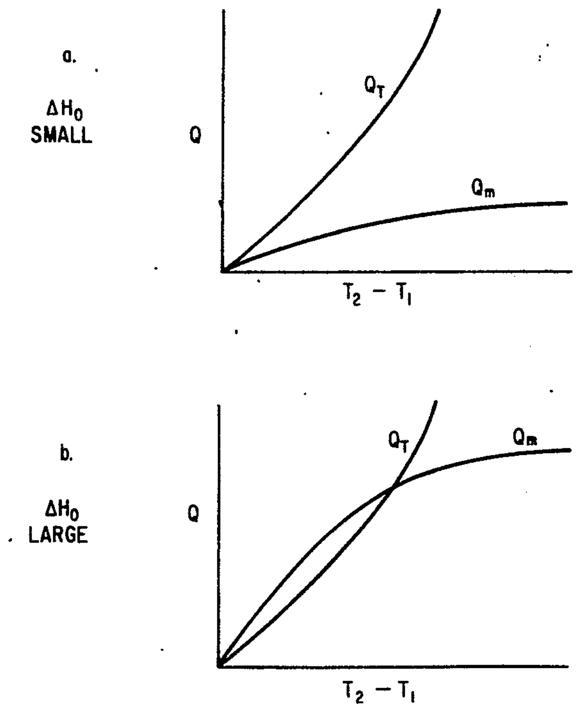


Fig. 7. Schematic illustrations of Q_m and Q_T vs T_2-T_1 for two cases:
a) Small excluded field; stable against small disturbances.
b) Large excluded field; unstable.

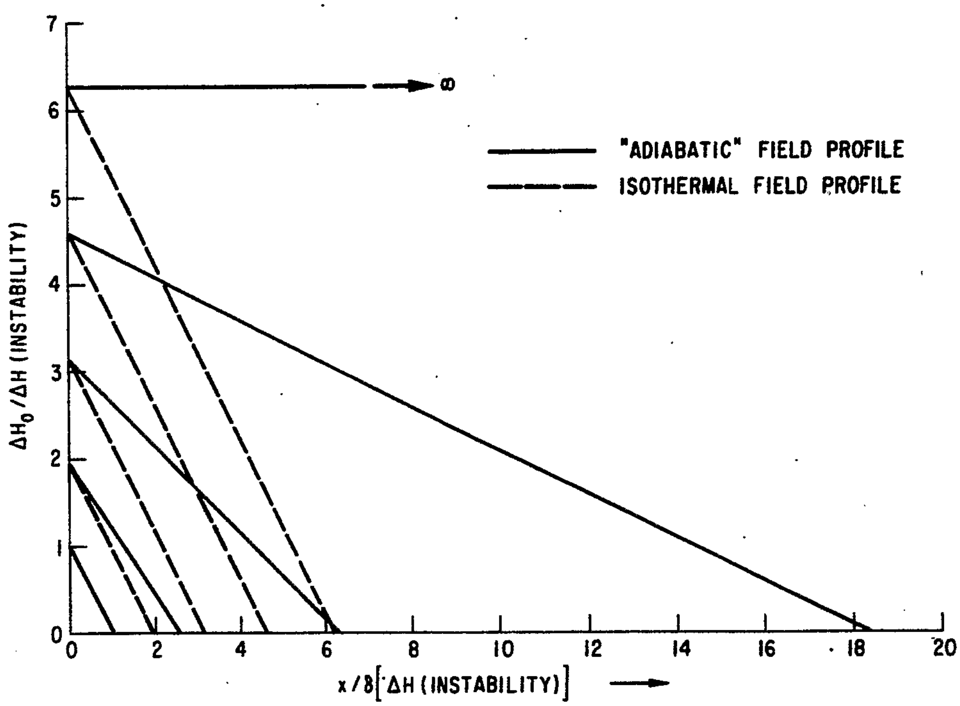


Fig. 8. Isothermal and quasi-adiabatic field profiles (following small disturbances) for a semi-infinite slab of a high field superconductor. The profiles were obtained as described in the text assuming $T_1/T_u = 0.2$.

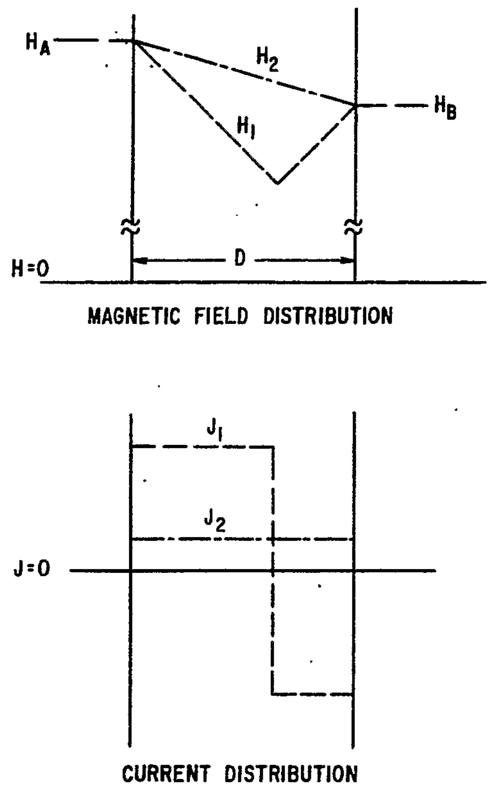


Fig. 9. The distribution of field and current density in a slab exposed to a field parallel to the plane of the slab ($H > H^*$) for a nonzero transport current. The two distributions shown are for the starting temperature, T_1, and for the highest temperature, T_2, for which the transport current can be carried as a supercurrent, i.e., the quench temperature. This is the current distribution used by Hancox[30] in his approximate treatment of a winding layer in a round wire solenoid.

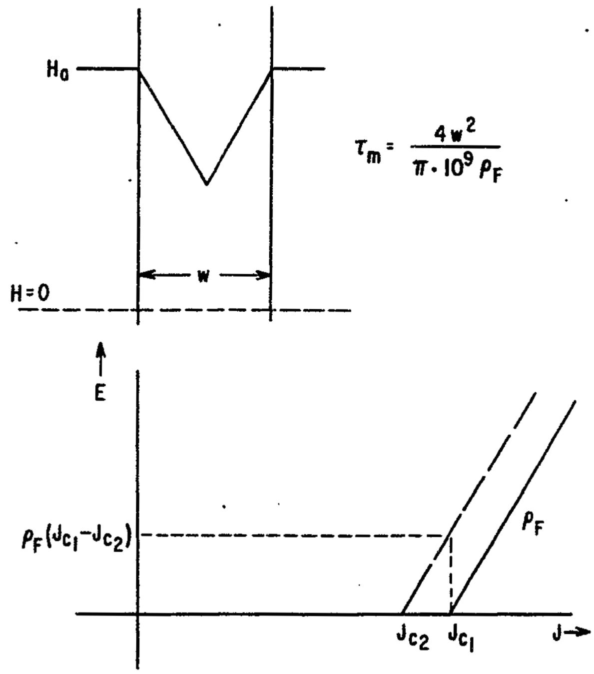
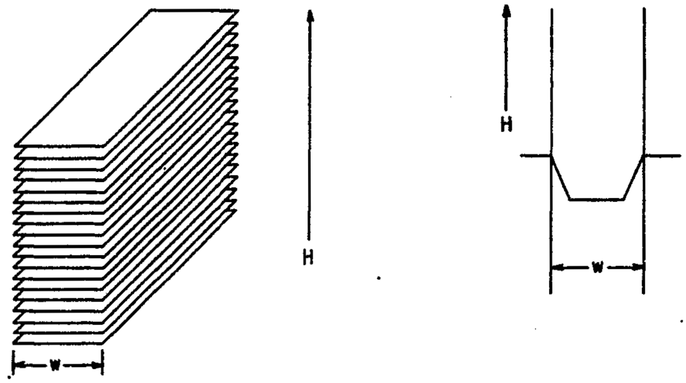


Fig. 10. For $H > H^*$, as illustrated, the magnetic time constant τ_m is given in terms of the slab thickness and the effective flux flow resistivity. In the lower portion the E-J relation is given for two temperatures; $T_2 > T_1$. The dashed lines illustrate the electric field and current density applicable for $t \ll \tau_m$ if the temperature is suddenly changed from T_1 to T_2. The local rate of heat generation is $E \cdot J$.



$$\frac{\partial H}{\partial X} = \mp \frac{4\pi}{10} \bar{J}_c$$

$$\bar{J}_c = \frac{d_s}{d_{tape}} \cdot J_s$$

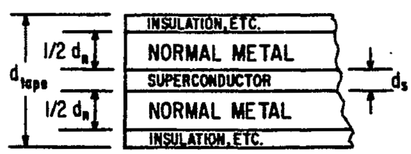


Fig. 11. In our analysis we replace a magnet winding wound from a wide thin composite tape with a stack of such tapes. When there is a component of magnetic field perpendicular to the plane of the tape the stack excludes field as would a critical state superconductor with a critical current density equal to $\bar{J}_c$. A simplified cross section of a tape is shown.

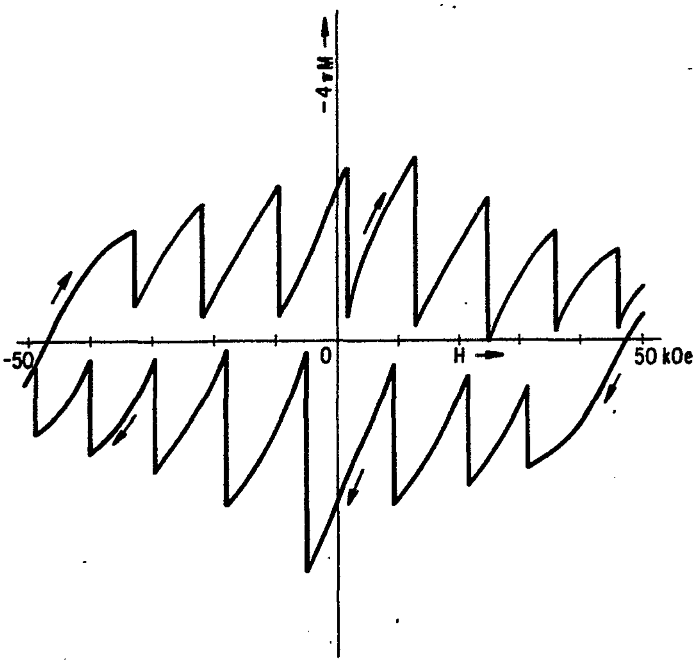


Fig. 12. A stack of disks punched from a thin Nb_3Sn-copper composite tape was exposed to a magnetic field perpendicular to the plane of the tape. The stack (1 in. high, ¼ in. diam) was relatively isolated from the liquid helium bath. The major flux jumps observed in the magnetization curve indicate that the system is not adiabatically stable.

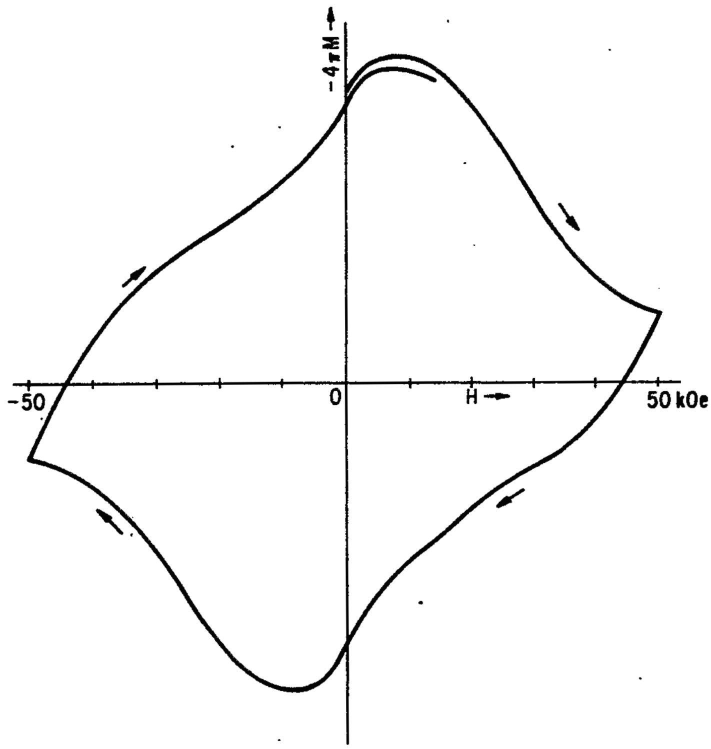


Fig. 13. A magnetization curve for a stack of disks similar to that of Fig. 12 except that very good thermal contact was made with a 4.2°K liquid helium bath. The absence of flux jumps indicates that we have achieved dynamic stability.