PRINCIPLES OF STABILITY IN COOLED SUPERCONDUCTING MAGNETS*

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INTRODUCTION

The behavior of a composite conductor is determined by the characteristics of the superconductor, the substrate which surrounds it and the characteristics of the cooling medium used to carry away the heat.

This paper considers the general case of a composite conductor taking into account the temperature drop within the superconductor itself, a thermal interface resistance between the superconductor and the substrate, and the cooling of the conductor by a coolant.

The approach taken consists of analyzing the steady state characteristics. Since the purpose of studying the stability of a conductor is to achieve steady state stability, the problem should be tractable by understanding the steady state behavior, with onset of instability being derived from the response of the coil in a quasi-steady manner to a transient triggering phenomenon.

In the following treatment the emphasis is on the physics of the phenomenon rather than the derivation of a set of equations that can accurately predict the terminal characteristics of a particular composite conductor.

FORMULATION OF EQUATIONS

Consider the general case of a conductor composed of superconductor (in many strands) and a cooled stabilizing substrate which completely surrounds each strand of superconductor.

The following assumptions will be made:

1) All the superconducting strands have equal properties and have the same shape and size.
2) There are no thermal gradients in the stabilizing substrate.
3) The heat generated within a superconducting strand is transferred to the substrate through a surface thermal contact resistance which is assumed to be uniform throughout.
4) There are no thermal gradients along the conductor so that we can deal with heat flow transverse to the conductor length.
5) The current density at any point within each superconducting strand is assumed to depend only on the local temperature. Also the current density is assumed to decrease to zero linearly with increasing temperature:

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$$j_s = j_{ch} \left[1 - \frac{T_s - T_b}{T_{ch} - T_b} \right] \quad , \tag{1}$$

where j_s = local superconductor current density,

j_{ch} = local superconductor current density at bath temperature,

T_s = local superconductor temperature,

T_b = bath or coolant temperature,

T_{ch} = temperature at which the current density goes to zero. (In the presence of whatever magnetic field exists.)

The temperature distribution within each superconducting strand is determined by heat conduction toward the superconductor-substrate interface and by the heat generation within the superconductor when a voltage exists across it. Without specifying the detail shape of the superconducting strands, we can write down the equation for their temperature distribution:

$$k \nabla^2 T_s = v \cdot j_s \quad , \tag{2}$$

where k = thermal conductivity (assumed constant),

v = voltage per unit length of conductor.

The right-hand side of Eq. (2) represents the heat generated per unit volume within the superconductor itself.

At the surface of the superconducting strand the temperature is assumed to be uniform and equal to T_w.

The total current carried by all the superconducting strands is:

$$I_s = n \int j_s \, dA_s \quad , \tag{3}$$

where n is the number of superconducting strands.

All the heat generated within a superconducting strand flows through the thermal contact resistance at the superconductor-substrate interface into the substrate:

$$v \cdot \left(\frac{I_s}{n} \right) = h_i P_i (T_w - T_{sub}) \quad , \tag{4}$$

where h_i = superconductor-substrate interface heat transfer coefficient,

P_i = perimeter of one strand of superconductor in contact with the substrate,

T_{sub} = temperature of the substrate.

All the heat generated within the conductor passes from the substrate to the coolant:

$$v \cdot I = hP(T_{sub} - T_b) \quad , \tag{5}$$

where I = total current in the conductor,

h = heat transfer coefficient at the substrate-coolant interface,

P = perimeter exposed to the coolant.

The voltage per unit length of conductor is determined by the current that flows in the normal substrate:

$$v = \frac{\rho}{A}(I - I_s) \quad , \tag{6}$$

where ρ = resistivity of the substrate,

A = cross-sectional area of the substrate.

It is very informative to put the equations into dimensionless form:

$$\frac{j_s}{j_{ch}} = 1 - \frac{T_s - T_b}{T_{ch} - T_b} \tag{7}$$

$$\frac{kA\,(T_{ch} - T_b)}{\rho\, I_{ch} j_{ch}\, r_w^2} \nabla'^2 \left(\frac{T_s - T_b}{T_{ch} - T_b} \right) = \left(\frac{vA}{\rho\, I_{ch}} \right) \left(\frac{j_s}{j_{ch}} \right) \tag{8}$$

where ∇' has been nondimensionalized with respect to the half width or half thickness of a superconducting strand r_w:

$$\frac{I_s}{I_{ch}} = \int \left(\frac{j_s}{j_{ch}} \right) \frac{dA_s}{A_s} \tag{9}$$

$$\left(\frac{vA}{\rho\, I_{ch}} \right) \left(\frac{I_s}{I_{ch}} \right) = \left(\frac{nh_i P_i A (T_{ch} - T_b)}{\rho\, I_{ch}^2} \right) \left(\frac{T_w - T_{sub}}{T_{ch} - T_b} \right) \tag{10}$$

$$\left(\frac{vA}{\rho\, I_{ch}} \right) \left(\frac{I}{I_{ch}} \right) = \frac{hPA(T_{ch} - T_b)}{\rho\, I_{ch}^2} \left(\frac{T_{sub} - T_w}{T_{ch} - T_b} \right) \tag{11}$$

$$\left(\frac{vA}{\rho\, I_{ch}} \right) = \frac{I}{I_{ch}} - \frac{I_s}{I_{ch}} \quad . \tag{12}$$

In the above equations

I_{ch} = critical current = $j_{ch}A_s$,

A_s = total superconductor cross section,

r_w = half thickness or radius of superconducting strand.

Let us now define the following variables:

$$J = j_s/j_{ch} \tag{13}$$

$$\theta = \frac{T - T_b}{T_{ch} - T_b} \text{ (with appropriate subscript s, sub, w)} \tag{14}$$

$$V = \frac{vA}{\rho\, I_{ch}} \tag{15}$$

$$\tau = I/I_{ch} \tag{16}$$

$$\alpha = \frac{\rho I_{ch}^2}{hPA(T_{ch} - T_b)} \tag{17}$$

$$\alpha_i = \frac{\rho I_{ch}^2}{nh_i P_i A(T_{ch} - T_b)} \tag{18}$$

$$r_{cs} = \sqrt{\frac{k(T_{ch} - T_b)A}{\rho\, j_{ch} I_{ch}}} = \frac{A}{I_{ch}} \sqrt{\frac{k(T_{ch} - T_b)}{\rho} \frac{A_s}{A}} \quad . \tag{19}$$

With these variables the equations reduce to:

$$J = 1 - \theta_s \tag{20}$$

$$\left(\frac{r_{cs}}{r_w}\right)^2 \nabla'^2 \theta_s = VJ \tag{21}$$

$$\left(\frac{I_s}{I_{ch}}\right) = \int J \frac{dA_s}{A_s} \tag{22}$$

$$V\left(\frac{I_s}{I_{ch}}\right) = \frac{\theta_w - \theta_{sub}}{\alpha_i} \tag{23}$$

$$V\tau = \frac{\theta_{sub}}{\alpha} \tag{24}$$

$$V = \tau - \left(\frac{I_s}{I_{ch}}\right) \quad . \tag{25}$$

Eliminating and rearranging the above equations we arrive at:

$$V = \tau - 1 + \alpha V\tau + \alpha_i V(\tau - V) + \int (\theta_s - \theta_w) \frac{dA_s}{A_s} \quad . \tag{26}$$

The quantity $\theta_s - \theta_w$ is the dimensionless temperature rise in the superconducting strands.

Making use of Eqs. (20) and (21) it can readily be shown that:

$$\nabla'^2 [(\theta_s - \theta_w) - (1 - \theta_w)] + V\left(\frac{r_w}{r_{cs}}\right)^2 [\theta_s - \theta_w - (1 - \theta_w)] = 0 \ , \tag{27}$$

which has a solution of the form:

$$\theta_s - \theta_w = (1 - \theta_w) + F\left[\left(\frac{r}{r_{cs}}\right)\sqrt{V}\right] \ , \tag{28}$$

where F[ ] denotes a functional relationship. At $r = r_w$, $\theta_s = \theta_w$ and the quantity $1 - \theta_w$ can be eliminated:

$$\theta_s - \theta_w = F\left[\left(\frac{r}{r_{cs}}\right)\sqrt{V}\right] - F\left[\left(\frac{r_w}{r_{cs}}\right)\sqrt{V}\right] . \qquad (29)$$

The quantity under the integral sign in Eq. (26) is simply the average value of $\theta_s - \theta_w$ over the superconductor area, which means integration over r, so:

$$\int (\theta_s - \theta_w) \frac{dA_s}{A_s} = G\left[\left(\frac{r_w}{r_{cs}}\right)\sqrt{V}\right] , \qquad (30)$$

where G[ ] represents a functional relationship. Substitution into Eq. (26) yields the final relationship:

$$V = \tau - 1 + \alpha V\tau + \alpha_i V(\tau - V) + G\left[\left(\frac{r_w}{r_{cs}}\right)\sqrt{V}\right] \qquad (31)$$

given the quantities α, α_i, and r_w/r_{cs} and the geometry of the strands so that the functional relationship G is defined, the curve of V vs τ is completely determined.

DISCUSSION

Equation (31) above shows very graphically the effects of heat transfer to the coolant, the interface thermal resistance, and the conductor size. Very simply the equation

$$V = \tau - 1 \qquad (\tau > 1) \qquad (32)$$

is the dimensionless voltage per unit length for a superconductor and a substrate, all current above the critical current ($\tau = 1$) flows in the normal substrate and produces a voltage proportional to the excess current over the critical value. As yet no account has been taken of the heating of the conductor as a whole, which is the product of the voltage times the current.

If the next term is added to Eq. (32):

$$V = \tau - 1 + \alpha V\tau \qquad (\tau > 1) . \qquad (33)$$

The additional term is indeed proportional to the dimensionless voltage times the dimensionless current $V\tau$, and the proportionality constant:

$$\alpha = \frac{\rho I_{ch}^2}{hPA(T_{ch} - T_b)}$$

is simply the temperature rise with all the current in the substrate divided by the temperature rise for the current density to go to zero in the superconductor.

The term $\alpha V\tau$ can be thought of as an additional voltage which appears across the superconductor due to imperfect cooling by the coolant. The quantity α is a measure of the cooling effectiveness.

The next term to be added is $\alpha_i V(\tau - V)$. Referring to Eq. (25) this term can be written as $\alpha_i V(I_s/I_{ch})$. In this form it is analogous to the $\alpha V\tau$ term, with the difference that the interface perimeter and heat transfer coefficient are used, and that since the temperature drop across the interface is proportional to the voltage times the superconductor current rather than the total current, the quantity τ is replaced by I_s/I_{ch}.

The constant of proportionality α_i is analogous to the quantity α except that the interface heat transfer coefficient h_i and total perimeter of contact nP_i are used.

The last term in Eq. (31) contains the quantity $(r_w/r_{cs})\sqrt{V}$. It is evident that if $V = 0$ there is no heating in the superconductor, and therefore $G(0) = 0$.

The effect of superconductor size and its interaction with other quantities is best revealed by examining the slope of the voltage-current characteristic at the critical current ($\tau = 1$) when voltage first begins to appear ($V = 0+$).

Taking the derivative of Eq. (31) with respect to V:

$$1 = \frac{d\tau}{dV} + \alpha\left[\tau + V\frac{d\tau}{dV}\right] + \alpha_i\left[(\tau - V) + V\left(\frac{d\tau}{dV} - 1\right)\right] + \frac{dG}{dV} \tag{34}$$

at $\tau = 1$ and $V = 0$ we can solve for $d\tau/dV$:

$$\frac{d\tau}{dV} = 1 - \alpha - \alpha_i - \frac{dG}{dV} \quad . \tag{35}$$

The derivative dG/dV is evaluated as follows:

$$\frac{dG[u]}{dV} = \frac{dG}{du} \cdot \frac{du}{dV} = \frac{1}{2}\frac{du^2}{dV} \cdot \frac{dG}{u\,du} = \frac{1}{2}\left(\frac{r_w}{r_{cs}}\right)^2 \frac{dG}{u\,du} \quad , \tag{36}$$

where

$$u = \frac{r_w}{r_{cs}}\sqrt{V}$$

$$\left.\frac{d\tau}{dV}\right]_{\tau=1} = 1 - \alpha - \alpha_i - \frac{1}{2}\left(\frac{r_w}{r_{cs}}\right)^2 \left[\frac{dG}{u\,du}\right]_{u=0} \quad . \tag{37}$$

The quantity $dG/u\,du$ is a function of u only, and as $u \to 0$ it takes on a numerical value (determined by the shape of the superconductor).

Taking the inverse of Eq. (37) we have:

$$\left.\frac{dV}{d\tau}\right]_{\tau=1} = \frac{1}{1 - \alpha - \alpha_i - \frac{1}{2}\left(\frac{r_w}{r_{cs}}\right)^2 \left[\frac{dG}{u\,du}\right]_{u=0}} \quad . \tag{38}$$

The general condition for stability against small voltage excursions is to have $dV/d\tau$ finite, which requires the denominator to be non-zero.

This leads to the general stability requirement that

$$\alpha + \alpha_i + \frac{1}{2}\left(\frac{r_w}{r_{cs}}\right)^2 \left[\frac{dG}{u\,du}\right]_{u=0} < 1$$

for stability against small disturbances at the critical current. Stability against large disturbances requires detailed knowledge of the complete V-τ curve.

Several illustrative curves will now be examined.

CONSTANT α (α_i, $r_w \to 0$)

The case for constant heat transfer coefficient h leads to the situation where α can be considered to be a constant for a particular geometry of conductor. Figure 1 (from Ref. 1) shows the curve of dimensionless voltage vs dimensionless current for various values of α.

Two distinct types of operation are possible, depending on the value of α. For $\alpha < 1.0$ no voltage appears until $\tau = I/I_{ch} = 1$ (the superconductor critical current). For $I > I_{ch}$ the voltage increases gradually with current. The characteristic is everywhere single-valued.

For $\alpha > 1.0$ the operation is more complicated:

$0 < \tau \le \frac{1}{\sqrt{\alpha}}$ — V = 0 all current in superconductor

$\frac{1}{\sqrt{\alpha}} \le \tau \le 1$ — double-valued operation $\begin{cases} V = 0^{*} \\ \text{or} \\ V = \tau \end{cases}$

$\tau > 1$ — all current in substrate ($V = \tau$) .

For this case of constant α, stable operation occurs for $\alpha \le 1.0$. For $\alpha > 1.0$ stable operation is limited to currents up to $I_{ch}/\sqrt{\alpha}$. In the region of current above $I_{ch}/\sqrt{\alpha}$, metastable operation with all the current in the superconductor is still possible up to I_{ch}, however the current can switch from the superconductor to the substrate should a large enough disturbance occur.

NONLINEAR HEAT TRANSFER CHARACTERISTIC

The nonlinear characteristic of the boiling curve of liquid helium can, to a first approximation, be represented by a region of constant heat transfer coefficient up to a surface temperature, T_m, which is the maximum temperature at which nucleate boiling can occur.[2] Above this temperature a transition to film boiling will occur, and to a first approximation we can assume a constant heat flux per unit area.

Under these assumptions the conductor terminal characteristics can be calculated. Figure 2 (from Refs. 3 and 4) shows the case where

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*Operation on the negative resistance part of the curve is unstable for constant current.

1. Z.J.J. Stekly and J.L. Zar, Avco Research Report AMP 210 (1965); IEEE Trans. Nucl. Sci. NS-12, No. 3, 367 (1965).
2. C.N. Whetstone and R.W. Boom, in Advances in Cryogenic Engineering (Plenum Press, New York, 1968), Vol. 13, p. 68.
3. Z.J.J. Stekly, Avco Research Report AMP 231 (1967); paper presented at the Intern. Cryogenic Engineering Conference, Kyoto, Japan, 1967.
4. Z.J.J. Stekly, R. Thome, E. Lucas, B.P. Strauss, and F. DiSalvo, Final Report NAS8-21037 (1968).

$$\theta_m = (T_m - T_b)/(T_{ch} - T_b) = 0.25 .$$

For $\alpha < 1$ no voltage appears until the current reaches the critical value ($\tau = I/I_{ch} = 1$), the voltage then rises gradually until the limit of nucleate boiling is reached. At this point all the current is expelled suddenly from the superconductor and transferred to the substrate. If the current is then reduced, a recovery occurs in which all or most of the current transfers back into the superconductor. (This cycle is shown in Fig. 2 for $\alpha = 0.5$.) For $\alpha > 1$ no voltage appears until the current reaches the critical value, then the current transfers abruptly out of the superconductor. Lowering of the current to a recovery value will again result in a return of all the current into the superconductor.

As can be seen in Fig. 2 there are three points which characterize the behavior of a particular conductor exposed to a magnetic field.

1) The critical current.

2) The transition from nucleate to film boiling from a condition where current is shared between the superconductor and the substrate. (This occurs above the critical current.)

3) The recovery from film boiling to nucleate boiling starting from a condition of all the current in the substrate.

A behavior map can be drawn of the different regions of operation for a particular behavior. This type of map is shown schematically in Fig. 3. In general, the map must take into account the variation of the critical temperature, the critical current, and the resistivity of the substrate with magnetic field.

The behavior of a typical superconductor has the following regions and curves:

a) I-H curve — represents the critical current of the superconductor.

b) Recovery curve — defined by the condition for the transition from film to nucleate boiling with all the current in the substrate.

c) Take-off curve — defined by the transition from nucleate to film boiling with current sharing between superconductor and substrate.

d) Stable zero resistance region — below the recovery curve and I-H curve.

e) Stable resistive region — above the I-H curve and below the recovery curve.

f) Metastable region — between the take-off and recovery curves: either above or below the I-H curve in which the conductor can be triggered into fully normal state from either a fully superconducting or current sharing condition.

g) Unstable region — operation is possible only with all the current in the normal conductor.

Figure 4 shows a portion of a stability map generated experimentally for a NbTi conductor.[5]

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5. Z.J.J. Stekly, E.J. Lucas, T.A. deWinter, B.P. Strauss, and F. DiSalvo, J. Appl. Phys. 39, 2641 (1968).

EFFECT OF THERMAL CONTACT RESISTANCE BETWEEN SUPERCONDUCTOR AND SUBSTRATE

Figure 5 shows the terminal characteristics for $\alpha = 0.5$ and for various values of α_i. As expected from Eq. (38), the slope at the onset of resistance becomes infinite at $\alpha + \alpha_i = 1$, or $\alpha_i = 0.5$ in the case shown.

For $\alpha_i > 0.5$ the voltage-current curves are double-valued below $I/I_{ch} = 1$. For this case below a certain value of current only the fully superconducting condition is possible, above this value of current the curve becomes double-valued.* Above the critical current, operation occurs with a significant fraction of the current in the substrate.

Specifically, for $\alpha_i = 5$ up to currents approximately 0.7 of the critical value the operation is single-valued. Between 0.7 and 1.0 of the critical current there is a metastable region where the voltage can remain zero or be triggered to the upper portion of the curve. At the critical current the voltage increases abruptly until about 95% of the current switches into the substrate. If the current is now lowered the current begins to transfer back into the superconductor and at about 0.7 of the critical current the voltage drops abruptly to zero.

Figure 6 summarizes the effect of interface thermal contact resistance. It shows on a logarithmic plot of α vs α_i the stable region (defined by $\alpha + \alpha_i \leq 1$) and the dimensionless voltage V_R and the dimensionless current τ_R at recovery to the superconducting state.

The region in the upper left of Fig. 6 (α large, α_i small) exhibits a behavior which is independent of α_i. Physically this region is poorly cooled to begin with and recovery occurs from the condition of all the current in the substrate, consequently at the beginning of the recovery there is no current in the superconductor and, therefore, no heat need be conducted across the superconductor-substrate interface.

It should be emphasized at this point that for the model of the conductor considered, which has the same current and temperature distribution along the conductor length, the current does not transfer from substrate to superconductor or vice versa along the conductor. Therefore, the electrical contact resistance is immaterial as long as the voltage drop per unit length is essentially the same in both the substrate and superconductor. It is the thermal contact resistance which plays a major role.

In practice, electrical contact is easier to make than thermal contact, and in conductors such as cables, wires embedded in strips, and soldered assemblies it is important to verify that good thermal contact has been achieved. This is best done by measuring the slope of the voltage-current curve at the critical current for a well-cooled ($\alpha \to 0$) sample.

EFFECT OF SUPERCONDUCTOR SIZE

The question of what effect the size of the superconducting strands has on the performance of a particular conductor is a very important one.

It is not the purpose of this paper to fully answer this question, but merely to point out that from a steady state stability point of view there are two mechanisms which have size dependence — the surface contact resistance, the temperature rise in the superconductor itself.

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*Operation on the negative resistance portion of the curve is unstable for a constant current source.

The total interface perimeter between the superconductor and substrate for round wires of diameter d is:

$$n\,P_i = n\,\pi\,d \quad .$$

If we desire a constant current in the over-all conductor then

$$I_{ch} = j_{ch}\,\frac{\pi\,d^2}{4}\,n \tag{40}$$

combining Eqs. (40) and (39) results in:

$$n\,P_i = \frac{4\,I_{ch}}{j_{ch}\,d} \quad . \tag{41}$$

For a superconductor with current density independent of size d, the perimeter increases inversely with d, which results in α_i proportional to d. However, in most superconductors the current density increases inversely as the square root of the size, so that the gain in perimeter is then only inversely as the square root of the diameter.

In a conductor with poor interface characteristics it would be expected that an improvement would result from going to smaller diameter strands if it could be assured that the same interface heat transfer coefficient would exist.

The temperature drop in the superconductor itself is more sensitive to size variations than the interface thermal contact resistance parameter α_i.

Figure 7 shows the results[4] of the dimensionless voltage-current calculations for a well-cooled ($\alpha = 0$) round wire. For $r_w/r_{cs} > \sqrt{8}$ the onset of resistance at the origin is abrupt and a double-valued region exists below the critical current.

As an example, let us consider a 0.020 in. (0.0508 cm) outer diameter wire with a single 0.010 in. (0.0254 cm) diameter NbTi strand at 50 kG which carries 60 A. We then have the following conditions:

$$\begin{array}{lll}
2\,r_w & = 0.0254 \text{ cm} & \\
A & = 1.52 \times 10^{-3} \text{ cm}^2 & \\
I_{ch} & = 60 \text{ A} & \\
P & = 0.16 \text{ cm} & \\
j_{ch} & = 1.3 \times 10^{5} \text{ A/cm}^2 & \text{(NbTi at 50 kG)} \\
k & = 2 \times 10^{-3} \text{ W/cm }^{o}\text{K} & \text{(NbTi)} \\
\rho & = 2.5 \times 10^{-8}\ \Omega\cdot\text{cm} & \text{(Cu at 50 kG)} \\
T_{ch} - T_b & = 2.6^{o}\text{K} & \text{(NbTi at 50 kG)} \\
h & = 1 \text{ W/cm }^{o}\text{K} &
\end{array}$$

The value of r_{cs} is:

$$r_{cs} = 0.00635$$

and

$$\frac{r_w}{r_{cs}} = 2 \quad .$$

According to the plot in Fig. 7 this is a significant effect on the terminal characteristics even if the conductor is well cooled ($\alpha \to 0$).

For round wires it has been shown[4,6] that the initial slope at the first appearance of resistance is (neglecting α_i);

$$\frac{dV}{d\tau} = \frac{1}{1 - \alpha - \frac{1}{8}\left(\frac{r_w}{r_{cs}}\right)^2} \quad . \tag{42}$$

Using the values given:

$$\alpha = 0.142 \quad .$$

If we take the ratio of $dV/d\tau$ taking into account the wire size to the value assuming $r_w \to 0$ then:

$$\frac{dV/d\tau}{[dV/d\tau]_{r_w \to 0}} = \frac{1 - \alpha}{1 - \alpha - \frac{1}{8}\left(\frac{r_w}{r_{cs}}\right)^2} = 2.4 \quad .$$

This is a very significant effect and points to the fact that in this particular conductor the strand size cannot be neglected as far as its effect on the terminal characteristics is concerned.

The characteristic conductor size can be written as:

$$r_{cs} = \sqrt{\frac{k(T_{ch} - T_b)}{\rho\, j_{ch} j_{sub}}} \quad , \tag{43}$$

where j_{sub} is the substrate current density I_{ch}/A.

This relationship is shown plotted for round wires in Fig. 8 for a composite NbTi conductor at 50 kG. The quantity $d_w = 2r_{cs}$ is the wire diameter at which the strand begins to affect the terminal characteristic. If $d_w = 2\sqrt{8}\, r_{cs}$ then the wire size is large enough to result in a jump in voltage at the critical current. Also shown on the plot are values for the ratio of copper area to superconductor area. Most commonly available high current density multistrand composites have ratios of copper to superconductor areas ranging from 3 to 1. The curve shows that depending on j_{sub}, wire diameters between 5 and 2.5 mils respectively would result in practically no effect on the terminal characteristics. On the other hand, strand diameters between 14 and 8 mils respectively would result in instabilities due to the strands themselves.

The actual numerical values used may be in error, especially the thermal conductivity, which has been assumed to be 2×10^{-3} W/cm °K, so that the actual numerical results must be interpreted very loosely.

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6. Z.J.J. Stekly, to be published.

CONCLUDING REMARKS

The steady state stability of composite superconductors has been reviewed taking into account cooling of the substrate, interface thermal contact resistance and temperature rise within the superconductor itself. Gradients along the conductor itself were not considered.

The general equations were derived and the effects of substrate cooling, interface thermal contact resistance and temperature rise in the superconductor were shown to affect the slope of the voltage-current curve at onset of resistance.

It should be re-emphasized at this point that the treatment in this paper is aimed at presenting relatively simple results so as to provide a maximum of physical insight into the behavior of composite superconductors rather than a detailed prediction of the terminal characteristics.

The assumption of constant heat transfer coefficient, and the assumption of very steep resistance rise at the critical current for the superconductor alone are both relatively crude approximations.

The effects of varying heat transfer coefficient can easily be taken account of in any single instance by estimating a voltage-current curve with constant heat transfer coefficient (constant α), computing the heat flux and then recomputing a new curve with the correct heat transfer coefficient at each value of heat flux. This procedure can be performed numerically or graphically until the results converge.

The approximation of the superconductor characteristic by a more complex model can also be done relatively easily; this, however, introduces more parameters, and it was felt that little additional physics would result from a more complex model.

The results derived are based on a steady state analysis of the terminal characteristics. Regions defined as fully stable, with all the current in the superconductor, are expected to be fully superconducting in reality.

However, in all metastable regions there exists the possibility of a triggering disturbance that will force the current out of the superconductor into the substrate.

For instance, in all cases considered one branch of the voltage-current curve is the $V = 0$ line extending up to the critical current. There is nothing in the analyses presented that indicates anything more than that in certain current ranges there are other operating points (usually with all or large fractions of the current in the normal substrate).

It can be concluded that if only one stable operating point exists then the conductor will operate there. If there is more than one operating point then it is necessary to study the triggering process.

It should be borne in mind that the aim of studying stability of superconductors is to achieve magnets that can operate reliably at a given current, and do not quench at the slightest mechanical or electrical disturbance. Since the desire is to achieve steady state stability, the conditions for achieving this stability as well as its limits should be obtainable from a study of steady or quasi-steady characteristics of the conductor.

Finally, one of the important results in this paper is that below a certain strand size, the temperature rise in the superconductor is very small ($r_w < r_{cs}$). For these conditions the temperature of the superconductor is essentially equal to that of the substrate. In order for superconductor generated instabilities to occur it is necessary to have a temperature rise in the superconductor. It can, therefore, be concluded that for $r_w < r_{cs}$ no superconductor generated instabilities or flux jumps can occur. If $r_w > \sqrt{8}\, r_{cs}$ the individual strands are unstable at their critical currents. In the region $r_{cs} < r_w < \sqrt{8}\, r_{cs}$ the possibility of collective interaction exists where strands may interact with each other.

The analysis presented in this paper is intended to explain in a quasi-quantitative way some of the experimental behavior of composite conductors. In doing so it points out the important variables which influence stability or the lack of it. Experimental work is necessary to verify (or contradict) the predicted characteristics of composites. It is hoped that the theory presented here can provide a framework for these experiments.

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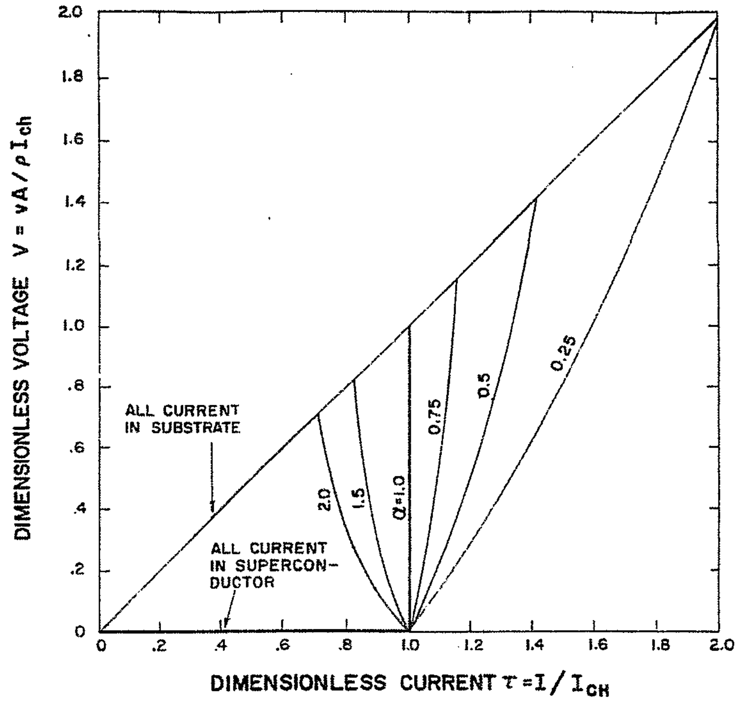

Fig. 1. Voltage-current characteristics for a stabilized superconductor-substrate combination.

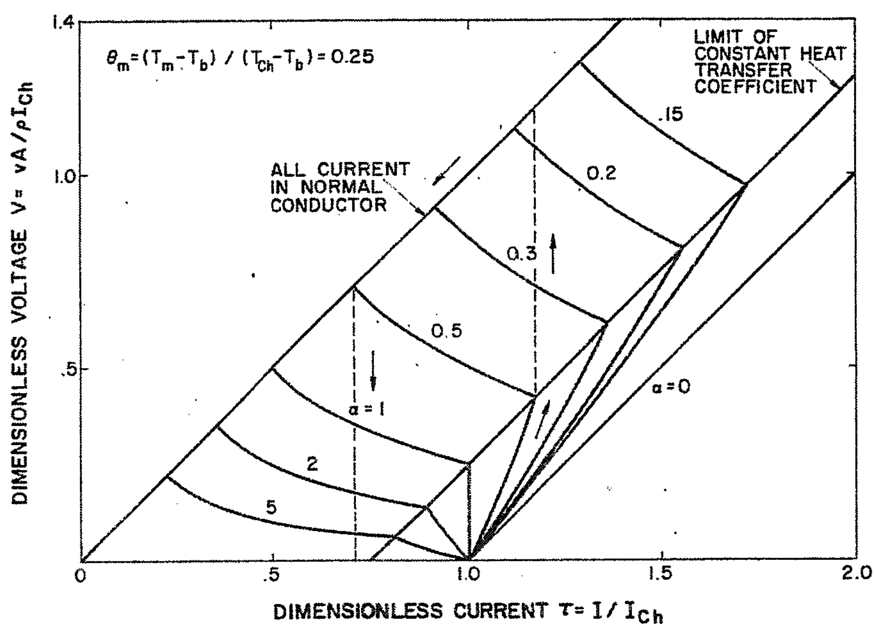

Fig. 2. Dimensionless terminal characteristics of a conductor taking into account through a simplified model the change in heat transfer characteristics resulting from a transition from nucleate to film boiling.

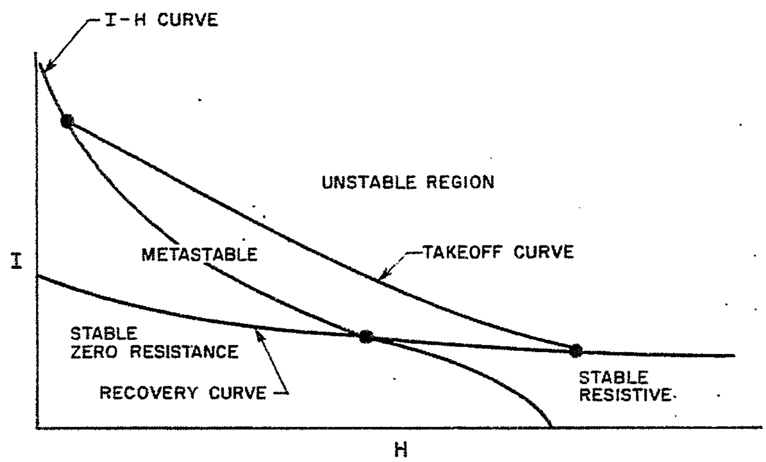

Fig. 3. Map indicating modes of behavior taking into account the nonlinear heat transfer characteristics of liquid helium.

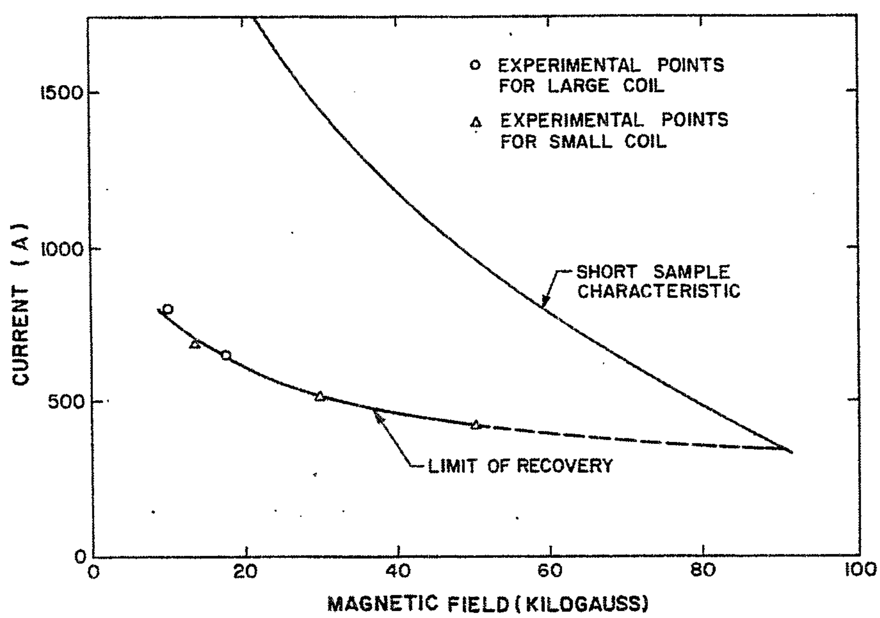

Fig. 4. Short sample and operating characteristics for the NbTi coil.

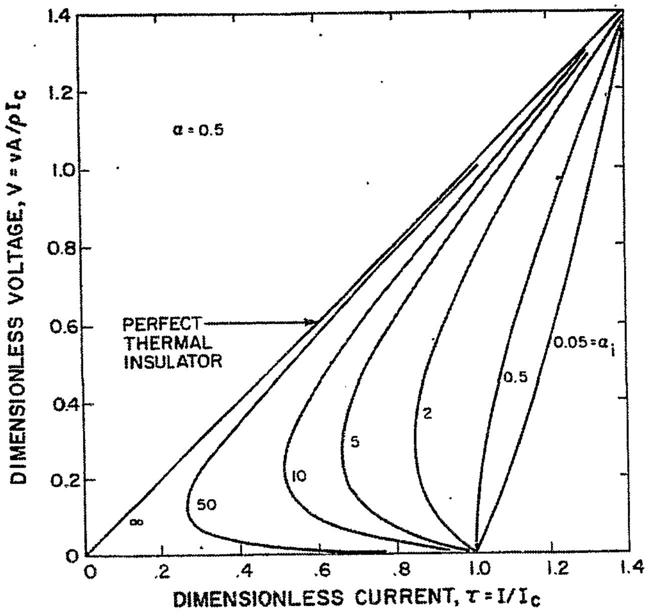

Fig. 5. Dimensionless terminal characteristics showing the effect of thermal contact resistance between the superconductor and substrate.

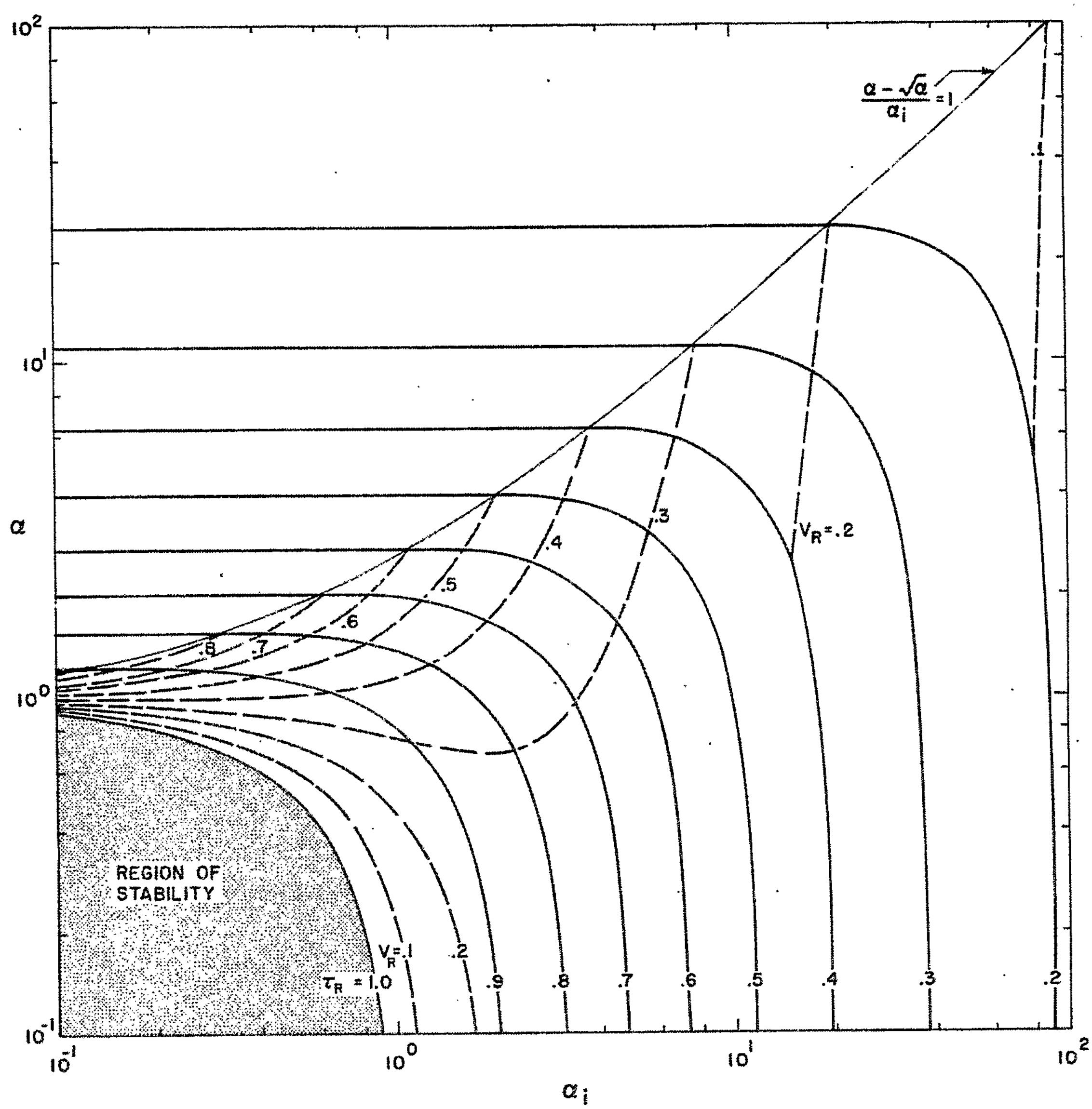

Fig. 6. Dimensionless recovery current τ_R and dimensionless recovery voltage V_R as a function of α and α_i.

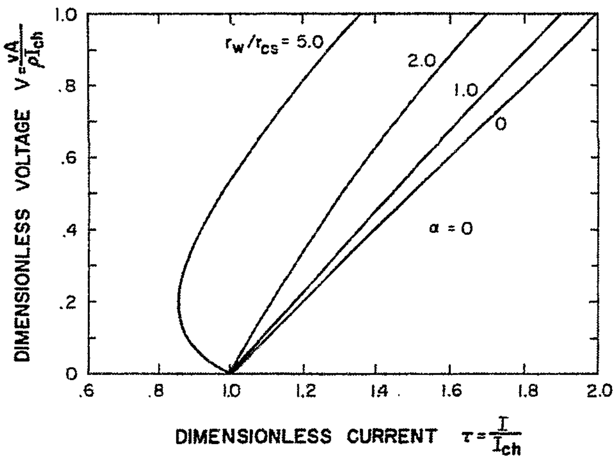

Fig. 7. Dimensionless terminal characteristics for a well cooled conductor ($\alpha \to 0$) as a function of the superconducting strands.

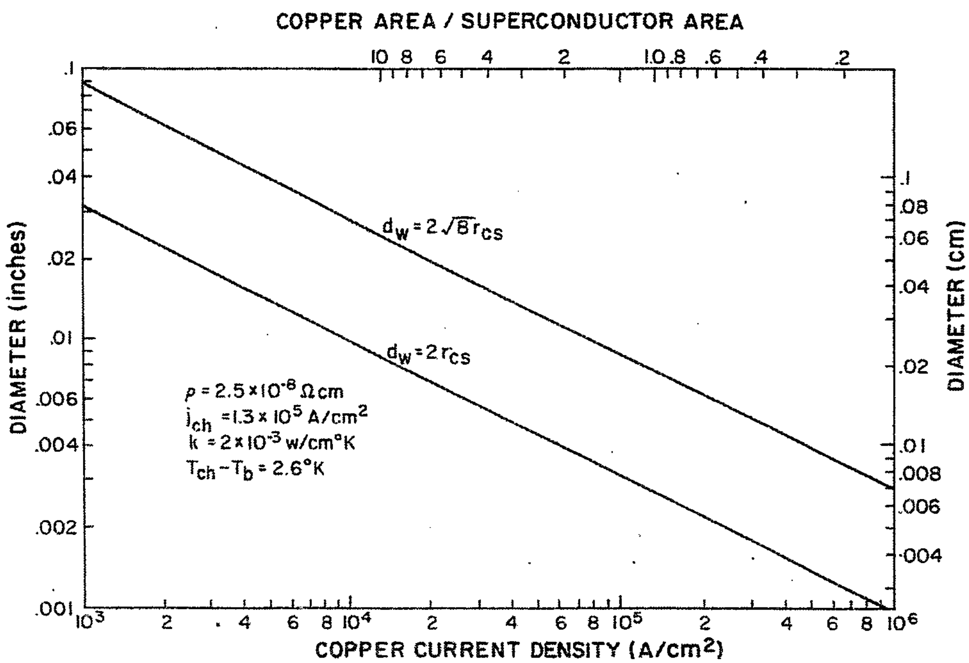

Fig. 8. Plot of $r_{cs} = \sqrt{\dfrac{k(T_{ch} - T_b)}{\rho\ j_{ch} j_{sub}}}$ NbTi conductors at 50 kG.

THE 1.8 TESLA, 4.8 M I.D. BUBBLE CHAMBER MAGNET*

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INTRODUCTION

The 1.8 T** magnetic field for the ANL 3.7 m chamber (Fig. 1) is provided by energizing coils (Fig. 2) wound from copper-clad, Nb48%Ti strip (Fig. 4) and immersed in liquid helium at 4.5°K (1.2 bar†) contained in a toroidal stainless-steel reservoir. The 4.8 m i.d. coils are located in a carbon steel yoke which forms a low reluctance return path for the magnetic flux. The use of iron reduces the ampere-turns needed to produce the required magnetic field. The carbon steel yoke and heavily copper-clad superconductor were chosen to give a reliable conservative magnet. The coil and cryostat are cooled by a separate closed cycle liquid helium refrigeration plant. A conventional power supply is used to energize the coil which is protected by low resistance, room temperature resistors. These can be switched across the coil terminals when required. The 3000 A, 10 V dc power supply is adequate to charge the magnet to design current in 2.5 hours.

The operating cost of the coil should be $400 000/year less than an equivalent copper coil because of the saving in electrical power. A conventional electromagnet of this performance with copper coils would have the same capital cost as the supermagnet ($3 000 000). It may be possible to replace the present coil with a 4.0 T coil which has the same winding space at some future date. The principal characteristics of the magnet are given in Table I.

The arrangement of the superconducting coils within the iron yoke and bubble chamber system is shown in Fig. 3. The inner and outer vacuum cans, and intermediate radiation shield can be seen in more detail in Fig. 4, where their position with respect to the windings is shown. The use of iron with a simple rectangular coil section results in a high field uniformity over the chamber volume (15%). Stray magnetic fields are reduced by the iron which also provides a sturdy mechanical foundation for the whole structure. The major portion of the frame is made up of castings weighing up to about 100 t.‡ Eddy current heating in the moving parts of the bubble chamber system is low because the field uniformity is high. This reduces the production of heat in the moving parts of the expander system and consequently the heat load on the hydrogen refrigeration system.

ENERGIZING COIL SYSTEM

The coil structure is split into two sections (Figs. 1, 3, and 4) to permit the beam of high energy particles to enter the chamber in a plane perpendicular to the line

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* Reprinted from Argonne National Laboratory Report ANL/HEP 6811 (1968), edited by C. Laverick.

** 1 T = 10 kG.

† 1 bar = 0.99 atm = 100 000 N/m^2.

‡ 1 t = 1000 kg.