SUPERCONDUCTING MAGNETIC DIPOLES*

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I. INTRODUCTION

This paper describes an approach to the design of superconducting magnetic dipoles. Numerical results are given for dipoles having several different geometrical arrangements of the conductors. The procedure applied here to the design of dipoles should be applicable to any magnet shape.

One of the few analytical solutions available in the design of superconducting magnetic dipoles is the cos θ distribution of the current density around a circle, or the corresponding distribution around an ellipse. However, the usefulness of the cos θ solution is limited by a number of other requirements. The magnetic field needs to be uniform to a high accuracy, approximately one part in a thousand. The conductors have considerable thickness and it becomes difficult to place them so that the cos θ distribution is achieved at all radii. For reasons of support and ease of winding, a surface made out of a series of planes may be more desirable than the elliptical surface required by the cos θ distribution.

It seems likely that some other more general approach to calculating the current distribution to obtain the dipole field is desirable which will permit considerable freedom in the geometrical arrangement of the current-carrying conductors. One such approach is investigated in this paper. In this approach, the field is specified at a certain number of discrete points on the median plane. Then for a given geometrical arrangement of rectangular conductors, the current is found in each conductor to give the desired field by solving a set of linear equations relating the currents and the field. The details of this procedure are given in Section V.

II. TWO-PLANE RECTANGULAR DIPOLE

Magnetic dipoles were investigated having three different geometries. Each dipole had a good field aperture of 9×18 cm^2. The geometrical arrangement of the conductors in each dipole is shown in Figs. 1 and 2, where one quadrant of the dipole is shown.

One dipole, shown in Fig. 1a, has a rectangular arrangement of conductors. The inner surface of the rectangle is at $\pm$ 10.5 cm and $\pm$ 6 cm, which allows 1.5 cm between the conductors and the good field region that can be used for the vacuum tank and supports for the conductors. There are ten conductors in each quadrant, each conductor being a region of constant current density, and each conductor has a thickness of 2.54 cm, which would correspond to two layers of a 0.5 in. ribbon.

The currents in the ten conductors for a 50 000 G field are given in Table I. The field variation in the median plane over $\pm$ 9 cm is $\Delta H/H = 1.6 \times 10^{-6}$. If we define the good horizontal aperture as a region where the field variation $\Delta H/H$ is less

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TABLE I. The current distribution for three magnetic dipoles having different geometries. $\Delta H/H$ is the fractional variation of the magnetic field in the median plane. Δx is the horizontal good aperture in centimeters.

Current in 10^5 A

Conductor Number	2-plane Rectangular	3-plane Elliptical	1-plane Planar
1	1.00025	1.16559	2.76034
2	1.01546	1.89295	2.99422
3	1.06907	2.27930	3.19995
4	2.26290	0.24313	3.21642
5	1.99581	1.05001	2.43819
6	0.46032	0.41540	0.84723
7	0.39499	0.65559	0.71688
8	0.25653	0.07148	0.25205
9	0.14740	0.14594	0.19254
10	0.04673	0.01953	0.05337
Total current, kA	865	794	1667
Maximum current density, kA/cm^2	59	59	42
$\Delta H/H$, y = 0	1.6×10^{-6}	1.6×10^{-5}	6.0×10^{-6}
Δx, y = 0	18	18	19
Δx, y = 2.5	18	15	21
Δx, y = 4.5	16	9	17
H_{max}, kG	56	57	82.5

than 2×10^{-3}, then the good aperture at y = 0 is x = ± 9 cm, at y = 2.5 cm, x = ± 9 cm, at y = 4.5 cm, x = ± 8 cm. Thus almost the entire region contained by the two-plane array has a satisfactory field. The maximum field at the conductors is 56 000 G or 12% higher than the field on the median plane. The largest current density is 59 000 A/cm^2 and the total current required in one quadrant is 865 000 A.

III. THREE-PLANE ELLIPTICAL DIPOLE

The second dipole is shown in Fig. 1b, and represents an attempt to approximate an ellipse more closely. The ellipse is shown by a dotted line. This dipole also allowed 1.5 cm for the vacuum tank and support and also had ten conductors in each quadrant with a thickness of 2.54 cm.

The currents in the ten conductors for a 50 000 G field are given in Table I. The field variation in the median plane over ± 9 cm is $\Delta H/H = 1.6 \times 10^{-5}$. The good horizontal aperture at y = 0 is x = ± 9 cm, at y = 2.5 cm, x = ± 7.5 cm, and at y = 4.5 cm, x = ± 4.5 cm. The good field region is close to that indicated by the ellipse with axis 4.5 × 9 cm shown in Fig. 1b. The total current required in one quadrant is 794 000 A. This is about 9% less than the current required for the two-plane array. However, one has to balance this against the smaller good field aperture that is obtained by the three-plane array. The maximum field at the conductors is 56 900 G, and the largest current density is 59 000 A/cm^2.

One can detect no important advantage for either the two-plane rectangular array or the more elliptical three-plane array. The choice between the two would be determined by the shape of the good field aperture required, and by consideration of the construction difficulties.

IV. PLANAR DIPOLE

The third dipole is shown in Fig. 2. This dipole has all the conductors in a single plane. Although clearly less efficient than the other two geometries, it may be worth considering because of the ease of winding this dipole. In particular, this type of dipole may be considered if the requirements on the uniformity of the field can be relaxed.

This dipole also allowed 1.5 cm for the vacuum tank, and the lower surface of the plane is 6 cm above the median plane. The required good field aperture is again 9 × 18 cm^2. There are ten conductors in each quadrant, with a thickness of 2.54 cm. The plane is 60 cm wide, or about 3.5 times as wide as the 18 cm of good horizontal aperture.

The currents in the ten conductors for a 50 000 G field are given in Table I. The field variation in the median plane over ± 9 cm is $\Delta H/H = 6 \times 10^{-6}$. The good horizontal aperture at y = 0 is x = ± 9.5 cm, at y = 2.5 cm, x = ± 10.5 cm, and at y = 4.5 cm, x = ± 8.5 cm. Thus almost the full rectangle of 9 × 18 cm^2 is a region of good field. The total current required in one quadrant is 1 667 000 A, which is about twice the current required in the rectangular geometry. The maximum field at the conductors is 82 500 G, or about 60% higher than the median plane field. The largest current density is 42 000 A.

The efficiency of this dipole could be improved if a leas uniform magnetic field were acceptable.

V. THEORY

The problem to be solved is: given a geometrical arrangement of N conductors, find the current in each conductor, C_n, in order to obtain a desired magnetic field in the median plane. Let us suppose that the magnetic field is specified at L points in the median plane, the points are designated by x_i, and the magnetic field at these points by H_i.

The equations for the C_n can be written in terms of $G_{i,n}$, which is the magnetic field at point x_i due to a unit current flowing in conductor n. The $G_{i,n}$ can be calculated from the geometry of the conductor arrangement. One can then write down the L equations

$$\sum_{n=1}^{N} G_{i,n} C_n = H_i \quad . \tag{1}$$

In principle one might solve Eq. (1) by making L and N equal. This, however, leads to mathematical difficulties in solving the equations. Solution of the equations requires a high degree of numerical precision, and the solution often gives currents too high to be acceptable. It is preferable to make L larger than N, and to solve the equations by a least squares procedure. One chooses the currents C_n by minimizing the expression

$$E = R^2 + q\, C^2 \quad , \tag{2a}$$

$$R^2 = \sum_{i=1}^{L} \sum_{n=1}^{N} (G_{i,n} C_n - H_i)^2 \quad , \tag{2b}$$

$$C^2 = \sum_{n=1}^{N} C_n^2 \quad . \tag{2c}$$

The term $q\, C^2$ is added in order to help rule out solutions with large oscillating currents. The parameter q is determined by trial, and a $q = 10^{-11}$ was used in the results listed in this paper.

Minimizing E in Eq. (2a) yields the equations for C_n,

$$(G' G + q I)\, C = G' H \quad . \tag{3}$$

Equation (3) is a matrix equation. G is the matrix $G_{i,n}$, G' is the transpose of G, C has the elements C_n, H has the elements H_i, and I is the identity matrix.

The solutions for C_n found from Eq. (3) were still not satisfactory, as so far only the field on the median plane has been specified. It was found that although the field on the median plane was equal to the specified field to a high degree of accuracy, the field off the median plane departed considerably from the desired field. Thus it was found desirable to specify the magnetic field at L_x points on the median plane and at L_y points along the y-axis off the median plane.

In order to specify the magnetic field along the y-axis, the following relations for the field along the y-axis may be used. Let us assume that the field in the median plane is given by the power series

$$H_y(x,0) = \sum_m b_m x^m \quad . \tag{4}$$

It follows from Maxwell's equations that the field along the y-axis is then given by

$$H_y(0,y) = b_0 - b_2 y^2 + b_4 y^4 - b_6 y^6 + \ldots , \quad (5a)$$

$$H_x(0,y) = b_1 y - b_3 y^3 + b_5 y^5 - b_7 y^7 + \ldots . \quad (5b)$$

In computing magnetic dipoles, H_y was specified along the y-axis. In computing quadrupoles, it would be more desirable to specify H_x along the y-axis as $H_y = 0$.

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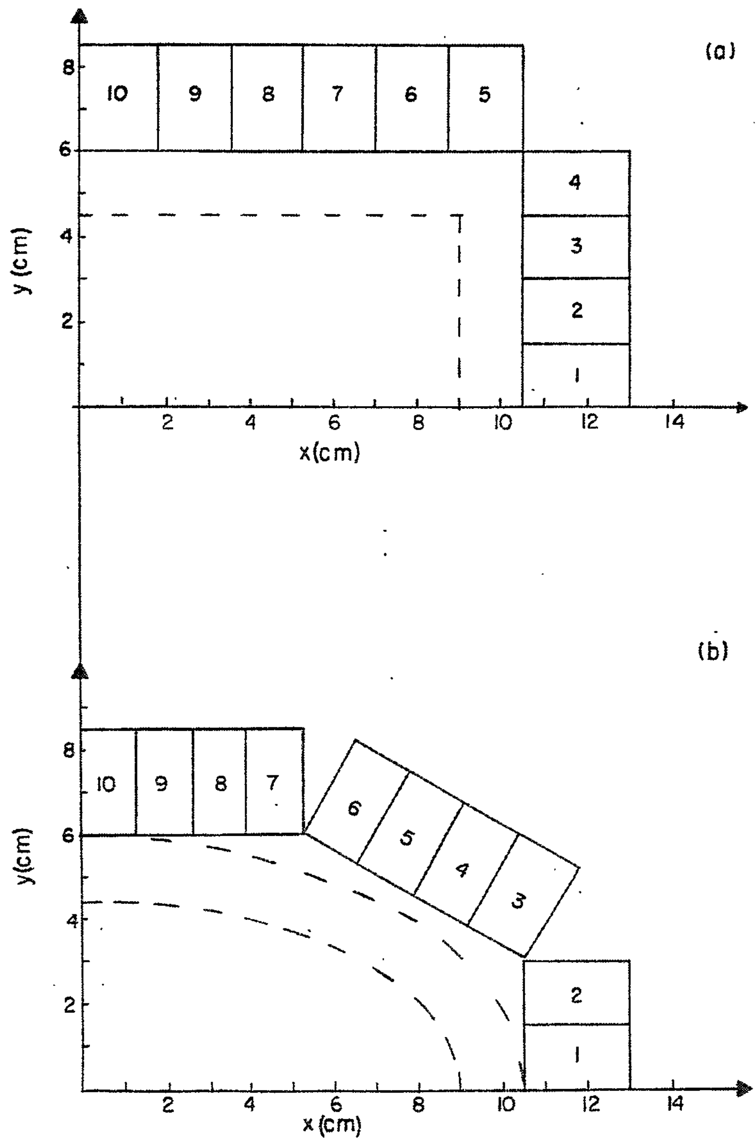

Fig. 1. The arrangement of the conductors in two magnetic dipoles, the two-plane rectangular dipole and the three-plane elliptical dipole.

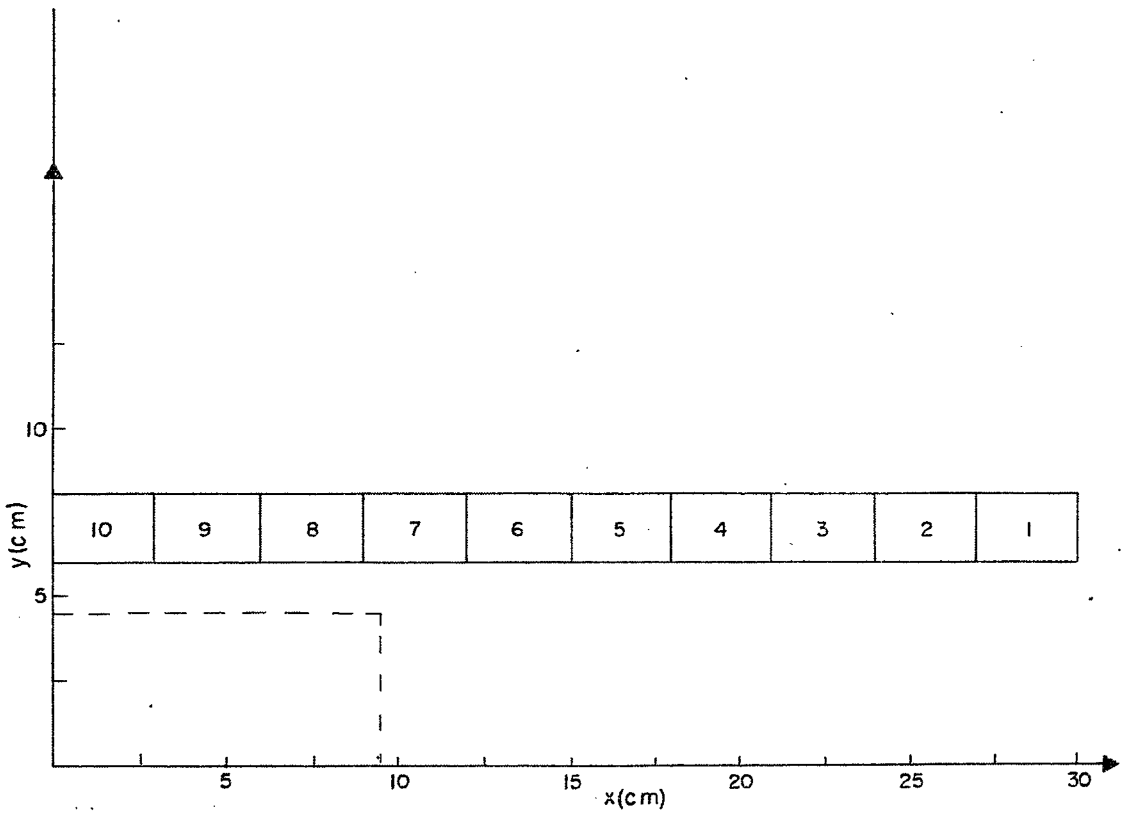

Fig. 2. The arrangement of the conductors in a planar magnetic dipole.