SUPERCONDUCTING SYNCHROTRONS

P.F. Smith
Rutherford Laboratory
Chilton, Berks., England

The subject will be discussed under the following headings:

I. Costs. Superconductor, engineering, refrigeration, power supply, and radius-dependent costs. Dependence on field, aperture, cycle time and ac loss. Separated function and combined function.

II. Ac losses. Basic theory; theory of losses in composite conductors; development of low-loss conductors.

III. Other design problems.

1. Basic features; coil configurations; typical parameters.
2. Eddy current heating; particle heating; stabilization and cooling; coil and cryostat materials; radiation damage.
3. Coil structure; forces; movement; winding accuracy; alignment.
4. Stray fields; shielding; remanent fields.
5. Operating current and voltage; protection; power supply; alternative power supply concepts.

IV. Specific designs. Results of exploratory studies of:

1. Conversion of Nimrod (7 GeV) to ~ 40 GeV.
2. Single stage 180 GeV ring, with 50 MeV injection.
3. Conversion of Nimrod to booster (5-25 GeV) for 100-200 GeV ring.

Most of the topics under Sections I, II, and III, were surveyed in a previous paper.[1] In the present review, attention will be concentrated on any progress made during the past year, and on plans and prospects for the immediate future.

In general, there has been little change in the over-all picture. Although there is widespread interest in this application, its long-term prospects are still rather uncertain compared with dc superconducting magnet applications, and the amount of effort devoted to it is correspondingly small. Paper studies, such as those described in Section IV, are instructive but lack realism until a suitable low-loss conductor is available for model magnet studies.

Development of a suitable conductor is, therefore, the overwhelming priority, and in this direction progress is fairly encouraging. It should not, as was originally feared, require the development of completely new fabrication techniques; theoretical studies of ac losses in composites composed of superconducting filaments in a metallic

1. P.F. Smith and J.D. Lewin, Nucl. Instr. Methods 52, 298 (1967).

matrix indicate that a low-loss conductor may be possible with only a moderate extension of existing manufacturing capability. We can anticipate, therefore, that within the next 12 months it should be possible to begin construction of realistic model magnets for tests under conditions of continuous pulsed operation.

I. CAPITAL COSTS

Accurate forecasts of costs are, of course, impossible at this early stage. It is, however, useful to have some simple approximate cost formulae as a basis for comparing exploratory designs, to indicate which cost components are dominant, and to set an economic limit to the ac loss.

For this purpose we are concerned only with those parts of the cost which are dependent on magnetic field, machine radius, and magnet aperture. To these must be added various fixed costs depending on the nature of the project (new accelerator, conversion of existing accelerator, new injector or booster required, etc.).

In each case the cost is estimated as a function of field H, radius R, and aperture, and divided by $3HR/10^7$ to obtain the cost in £/(GeV/c). It is simplest to consider initially only the bending field and discuss separately the additional cost of providing focusing. These estimates differ only slightly from those in Ref. 1, but dependence on aperture is now included. Based on experience of typical designs, the aperture is now assumed to be circular rather than elliptical (see Section IV).

1. Superconductor

Assume a distribution of current $I_1 \sin\theta$ (A/cm) around a circular aperture, giving a field $H = 0.2\pi I_1$ (G). Let r be the mean radius of the winding given by

r = beam aperture radius a
+ thickness of cryostat and coil former
+ half of coil thickness .

(Typically, for example, we might expect that $r \approx a + 2$ cm.)

Then the amount of superconductor required, allowing an additional 15% for magnet ends, is 40 HRr (A·cm) or 1.5×10^8 r (A·cm/GeV). Present cost of NbTi conductors is 4×10^{-10} H (£/A·cm) up to H ~ 80 kG; more optimistically we might guess that this could eventually be halved for large quantities. The cost range is therefore:

$$\text{Superconductor cost} = 0.06\ rH \text{ to } 0.03\ rH \quad (\pounds/\text{GeV}) \quad . \qquad (1)$$

2. Engineering

Cryostat costs are at present usually in the region £0.3 S, where S (cm^2) is the coil surface area. The coil construction costs are difficult to estimate and are here arbitrarily assumed to be proportional to the quantity of superconductor. Using costs experienced in recent dc magnet projects, and bearing in mind that design and drawing costs should be negligible for a large number of identical units, we guess the following cost range:

$$\text{Magnet engineering cost} = \begin{array}{l} 1000\ r + 8 \times 10^7\ r/H \\ \text{to } 500\ r + 4 \times 10^7\ r/H \end{array} \quad (\pounds/\text{GeV}) \quad . \qquad (2)$$

For example, for a 200 cm long, 60 kG magnet with a 5 cm beam aperture radius, this formula gives an engineering cost of £6000 to £12 000.

3. Refrigeration

Consider first the requirements other than the ac loss. There are contributions from conduction and radiation losses, current leads, joints, eddy currents, and particle heating, as estimated in Ref. 1. Assume that with care these can be limited typically to ~ 3 W/m of circumference for r ~ 6 cm, and that they are proportional to r. Costs of a variety of multimagnet refrigeration schemes have been estimated by Strobridge et al.[2] and it has subsequently been suggested that costs a factor ~ 2 lower might be achieved.

We arbitrarily assume subdivision into 1200 W units, allowing a factor 2 for transfer losses, at a cost, including distribution, of £3 × 10^5 to £6 × 10^5 per unit. This gives a basic refrigeration cost of 0.5 to 1 × 10^8 r/H (£/GeV).

Now assume an ac loss of P (W/m) and increase the size of each unit accordingly. At an incremental cost of £100 to £200/W the additional cost is 2P to 4P × 10^7/H (£/GeV). The total estimated cost range is thus

$$\text{Refrigeration capital cost} = (r + 0.4\ P)\ 10^8/H \quad \text{to} \quad (0.5\ r + 0.2\ P)\ 10^8/H \qquad (\text{£/GeV}) . \tag{3}$$

Thus for r ~ 8 cm, P can be ~ 20 W/m before the cost of refrigeration equipment is doubled.

4. Power Supply

Assuming a uniform field up to radius r, and equal stored energies inside and outside the coil, the total is approximately $r^2H/2$ J/GeV. Cost of conventional power supplies is proportional to the peak rate of energy transfer, and is usually about 0.04 £/J for the typical 1 sec rise time. Costs may be proportionately lower at very high stored energies, and because no dc power is required for "flat-top" conditions; on the other hand higher costs may be necessary to improve reliability. This suggests the cost range:

$$\text{Power supply cost} = 0.024\ r^2H \quad \text{to} \quad 0.012\ r^2H \qquad (\text{£/GeV}) \tag{4}$$

for a 1 sec rise time T, and proportional to 1/T in other cases.

5. Radius-Dependent Cost

In addition to the magnet tunnel cost, this includes such items as site preparation, magnet installation, vacuum system, site power, etc. In Ref. 1 it was suggested that about £30 million of the 300 GeV estimate would decrease linearly with radius, which is equivalent to 12 × 10^8/H (£/GeV). This may, however, be too optimistic, particularly if substantial reductions in magnet tunnel cost are possible. We allow for this by assuming:

$$\text{Radius-dependent cost} = 12 \times 10^8/H \quad \text{to} \quad 6 \times 10^8/H \qquad (\text{£/GeV}) . \tag{5}$$

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2. T.R. Strobridge et al., National Bureau of Standards Report 9259 (1966).

There is also the further complication that the tunnel radius will not usually reduce in proportion to the bending radius, since the straight section length is fixed by other considerations. In comparing specific designs, therefore, it is more realistic to take the radius-dependent cost to be £2000-£4000 per meter of circumference.

6. Optimum Values

The occurrence of terms proportional to H and 1/H leads to a cost-optimum field. The optimum costs are shown in Table I, using the means of the preceding cost ranges, for three typical values of r. The optimum is rather flat, and subject to the uncertainties in costs, so that the figures cannot be taken very seriously. Nevertheless, the general trends are of interest; firstly that the typical optimum fields 40-60 kG conveniently coincide with the field range desirable from practical viewpoints; secondly that all five items tend to be of the same order of cost; and thirdly that for larger apertures the power supply requirements begin to be unreasonably large.

TABLE I

"Optimum Costs, Without Focusing

Item	Mean of Cost Range (£/GeV)	Optimum costs in units £1000/GeV (for 3 sec cycle)		
		r = 10 cm H = 37 000 G	r = 7 cm H = 45 000 G	r = 4 cm H = 60 000 G
Superconductor	$0.045\ rH$	17	14	11
Power Supply	$0.018\ r^2H$	67	40	18
Engineering	$750\ r + 6 \times 10^7\ r/H$	24	15	7
Refrigeration	$1.5 \times 10^8\ r/H$	40	23	10
Radius-dependent items	$9 \times 10^8/H$	24	20	15
Total of above items (N.B. Constant cost items not included)		172	112	61

7. Cost of Focusing

Quadrupoles have approximately the same costs per unit length as bending magnets for the same field and aperture, with the exception that the power supply cost, proportional to stored energy, is only half as great. Thus, if the ratio of quadrupole length to bending magnet length in a separated function lattice is q, the above power supply cost is multiplied by $(1 + q/2)$, and the other costs are multiplied by $(1 + q)$. To a good approximation, therefore, we can assume the optimum to be unaffected, and all costs simply increased by $\sim (1 + 0.9\ q)$. In Section IV it will be found that typical values of q are between 0.15 and 0.3.

For a combined function machine the cost changes are more complicated, and not uniformly distributed. Simple approximations suggest that, in an equivalent combined function lattice, the over-all cost is multiplied by ~ (1 + 0.4 q). It thus follows that a separated function machine is theoretically dearer than a combined function machine by a factor ~ (1 + 0.5 q), i.e. ~ 10-15%. This cost difference, however, makes no allowances for the greater engineering complexity of the asymmetric separated function magnets.

8. Comparison with Conventional Magnets

Assuming r = a + 2 cm, the r = 7 figures in Table I (increased by 20% to allow for focusing), may be compared with the corresponding costs for a conventional synchrotron of 5 cm aperture radius, which amount to about £150 000/GeV. Specific designs, such as those in Section IV, suggest that, for a given energy and number of particles, the required aperture would in fact be smaller than in a conventional machine, resulting in a further cost gain. The economics of the superconducting system would be even more convincing if reductions in the power supply and refrigeration costs become possible. The possible cost gains are, of course, much greater in the case of conversions of existing accelerators, since many of the fixed cost items (site, experimental areas, injector, etc.) are absent.

Comparative running costs have not yet been estimated, and no further thought has been given to combined superconducting + conventional systems.

II. AC LOSSES

1. Objectives

High field superconductors have an irreversible magnetization curve. Changes in field must therefore generate heat, and the heat released per unit volume is proportional to the superconductor width perpendicular to the field. Using 0.025 cm wire, the dissipation in a typical 60 kG, 10 cm aperture diameter synchrotron magnet, with a 3 sec cycle time, would be about 500 W/m length of magnet. A similar or higher loss would be expected using Nb_3Sn tapes. This is obviously too high, both from economic and magnet design viewpoints, and although it is still not clear exactly how much reduction will be necessary, it appears that for most purposes a limit of 10-20 W/m would be a desirable objective. We are, therefore, faced with the problem of reducing the loss by a factor 25 to 50.

Three ways of doing this have been considered.

a) Reduction of average strand diameter to 5-10 μ (~ ¼ to ½ mil).

b) Thin layers of superconductor shaped to follow the field lines.

c) Increase current density by several orders of magnitude, so that the current is confined to thin surface layers (see below).

Methods b) and c) do not look feasible at present, but a) should be possible using existing manufacturing techniques in which a number of parallel strands are drawn down in a supporting matrix. Ideally the matrix should be an insulator, but a conducting matrix may also be suitable provided the strands are sufficiently twisted or transposed. Some aspects of the theory of this will now be discussed.

2. Theory

Consider first the usual calculation of the ac loss for a slab of superconductor (Fig. 1). Magnetization current J_c (A/cm^2) is induced by a changing external field, and completely fills the material when $H \simeq Jd/2$ (which will be for most of the cycle in the case of thin wires and high fields).

The loss/unit volume at any point is simply the current density times the local electric field (proportional to the magnetic flux crossing the element per unit time). Integrating this over the whole slab one obtains for the total energy released:

$$Q(J) = H(G) \times J_c \times V(\text{volume, cm}^3) \times d(\text{cm}) / 4 \times 10^8 \tag{6}$$

$$= H \times \left[\begin{array}{c}\text{A·cm of}\\ \text{superconductor}\end{array}\right] \times d \left[\begin{array}{c}\text{wire}\\ \text{diameter}\end{array}\right] / 4 \times 10^8 \quad . \tag{7}$$

More exact estimates for a complete coil must take into account the spatial variation of H, the variation of J_c with H, the transport current, and the shape of the conductor cross section, as discussed in Ref. 1. For small test coils, allowance must also be made for the zero ac loss below H_{c1} (which may be in the region ½ to 1 kG).

For coils wound from wide tape, particularly single layer windings, a further correction is necessary for field distortion, and for incomplete penetration of the flux during most of the cycle. Typically these two effects can multiply formula (6) by a factor 0.05 to 0.1, but the loss is usually still higher than in corresponding coils of 0.010 in. wire.

When the penetration depth is p, the above formula is multiplied by $(p/d)^2$, and since $p \propto H/J_c$, it follows that $Q \propto 1/J_c$, showing the possibility of decreasing Q by increasing J_c, as mentioned above.

Now consider the loss in a multifilament conductor. Fig. 2(a) shows two strands of superconductor embedded in an insulator. The two strands are independent (ignoring for the moment the problem of the eventual end corrections), and separate magnetization currents flow in each strand as shown. The ac loss is given by J_c × the local electric field, which is approximately proportional to the diameter of the strand.

In Fig. 2(b) the insulator is replaced by a normal conductor. If the resistance of the available path between the strands is sufficiently low, or if the rate of change of field is sufficiently high, the magnetization currents will be driven round the entire composite as shown. The ac loss will still be given by J_c × the local electric field, but the latter is now proportional to the distance between the strands.

For a lower rate of change of field, or a higher resistance path between strands, an intermediate state may exist in which some of the magnetization current is confined to the strands and the rest crosses the matrix [Fig. 2(c)].

In general, by providing sufficient twisting or transposition of the strands, a state of this type can be produced in which the majority of the magnetization current is confined to the strands, and the ac loss is then closely proportional to the filament diameter, as required. The criterion is that the twist pitch must be substantially less than a critical length ℓ_c given by

$$\ell_c \sim 10^8 \lambda J_c d \rho / \dot{H} \quad ,$$

where $\dot{H}$ is the rate of change of field in G/sec, ρ the matrix resistivity in Ω·cm, d the filament diameter in cm, J_c the filament current density in A/cm², and λ is a space factor. This is discussed in more detail in the paper on intrinsically stable conductors in these Proceedings.[3]

3. P.F. Smith, these Proceedings, p. 913.

With high resistance alloys, and $\dot{H} \sim 60$ kG/sec, it may be possible to achieve the required twist pitch. With copper as the matrix material the twist pitch appears to be impracticably low; if required, however, copper may be incorporated in the composite provided it does not directly link two superconducting filaments.

Initial composite diameters in the region 0.010 in. are envisaged, which could then be insulated and formed into multistrand cables to carry higher currents. These will probably have to be transposed rather than twisted cables, because of the magnitude of the field gradient across the conductor. An approximate theory of this effect indicates that twisting should nevertheless be adequate in the basic 0.010 in. composite.

Even if composites with an insulating matrix can be developed, a certain (at present uncalculated) amount of transposition or twist will be necessary because of the end connections. This is one of the difficulties in using a subdivided Nb_3Sn tape, since only one twist/turn would appear to be possible.

3. Experiments

The basic theory of losses in hard superconductors is well established, and amply verified experimentally to better than a factor of 2. Small coils of NbZr wire and Nb_3Sn tape tested at the Rutherford Laboratory at frequencies between 4 Hz and ¼ Hz have shown losses consistent with theory, and we regard further confirmation as unnecessary.

Our interest is therefore concentrated on the behavior of composites. A variety of samples containing NbTi filaments in a resistive matrix are being tested. Magnetization measurements have been made on small coils as a function of dH/dt, and the proportion of magnetization current in the strand and in the matrix are essentially in accordance with theory. The effects of twisting are also in accordance with theory. Some ac loss measurements are also being made using these composites.

It is proposed to extend this work to the required ½ or ¼ mil filament sizes. If large quantities can be produced satisfactorily and economically, such a material should be suitable for synchrotron magnets, and a more thorough and larger scale program of tests may be attempted.

III. OTHER DESIGN PROBLEMS

It is clearly not possible to make a realistic study of the majority of magnet design problems until the dominant problem of the ac loss is solved, and a clearer picture is obtained of the electrical and mechanical characteristics of the conductor likely to be used. Accordingly, although there has been some crystallization of ideas during the past year, there is relatively little to add to the remarks in Ref. 1.

1. Basic Parameters

It is now generally felt that separated function magnets will almost certainly be preferred to combined function magnets. Although the latter theoretically provide the most compact and economical system, the difference is usually fairly small (typically 10-20%, as explained above), and is offset by the greater simplicity and flexibility of the design, engineering, and commissioning of separate bending and focusing units.

The coil cross section could either consist of some convenient approximation to the overlapping ellipses configuration, or current blocks spaced to approximate the sin θ or sin 2θ circumferential distribution. The aim, of course, is to achieve uniformity of field or field gradient, and to minimize the peak field on the windings, with a shape which is reasonably straightforward from an engineering viewpoint. A high current density is essential for a compact and economic design, and a value in the region 30 000 to 50 000 A/cm^2 is usually assumed, giving a coil thickness of about 2 cm.

Typical optimum lattice designs require an aperture radius in the range 3 cm to 6 cm, and the aperture tends to be more nearly circular than in conventional machines. An important question, particularly for small apertures, is how much should be allowed for the difference between aperture radius and inner radius of winding. A figure of ~ 1 cm is usually taken, on the assumption that the beam pipe need not be at room temperature.

Table II shows a revised list of typical parameters for bending magnets of 3 cm and 6 cm usable aperture radius.

TABLE II

Typical Parameters for Bending Magnets

Beam aperture radius	6 cm	3 cm
Coil internal radius	7 cm	4 cm
Maximum winding thickness	2 cm	2 cm
Coil current density	48 000 A/cm^2	48 000 A/cm^2
Magnetic field (uniform)	60 kG	60 kG
Magnetic bending radius	55 cm/GeV	55 cm/GeV
Total magnet length	3.4 m/GeV	3.4 m/GeV
A·cm of superconductor	3.5×10^8/m	2.2×10^8/m
Stored energy/meter	0.57 MJ	0.22 MJ
Stored energy/GeV	1.9 MJ	0.75 MJ
Rise time	1 sec	1 sec
Cycle time	3 sec	3 sec
Refrigeration requirements at 4°K:		
a) Cryostat losses, current leads, eddy currents, particle heating	4 W/m	2.5 W/m
b) Ac losses (filaments 6×10^{-4} cm diameter)	20 W/m	12 W/m

2. Other Heating Effects

It was shown in Ref. 1 that eddy current heating in any normal metal in a composite conductor will be small provided that the diameter of the strands of composite is in the region 1 mm or less. To prevent excessive heating by high energy particles, the dose must be kept below 10^4 rad/h, and this means that particular care must be taken to ensure that the extraction loss is sufficiently low and not concentrated in a small region of the circumference.

It is hoped that stabilization will be unnecessary in a filamentary composite, so that the coil can be fully insulated, and impregnated for strength and rigidity, and cooled on the surface only. For an average thermal conductivity of $\sim 10^{-3}$ W/cm ^{o}K, the ac loss and other heating effects would cause a temperature rise of order 1^{o}K. This would presumably be only just tolerable, and some assessment is needed of the possibilities of insulators with a high thermal conductivity.

Eddy current heating appears also to necessitate the use of insulating materials for the cryostat and coil former. A preliminary assessment of this problem would also be of interest. With all these materials (and, to a lesser extent, the superconductor itself) the complication of long-term radiation damage must be taken into account.

3. Forces

For a thin winding, the total outward force on the coil is $aH^2/3\pi$ dyn/cm/length, where a is the radius of the coil (this is misprinted in Ref. 1). For the magnet dimensions envisaged, this force is smaller than that encountered in many existing or proposed dc magnet projects. Nevertheless there is obviously some anxiety about the rigidity and long-term reliability of the coil under continuous pulsed operation. Stresses arising from differential thermal contraction could present an even greater problem, with the mixed use of metallic and nonmetallic materials.

A related problem is that of defining and maintaining the coil position within the cryostat, to enable the required accuracy of magnet alignment to be achieved.

These problems are unlikely to be studied seriously until the first model magnets are designed.

4. Stray Fields

The field at a distance d (cm) from an unshielded bending magnet (central field H, mean winding radius r) is approximately $H(r/d)^2$. This is, of course, much higher than the stray field usually encountered in an accelerator magnet tunnel, and there is at present some disagreement as to whether magnetic shielding will be necessary or not. However, the use of concentric superconducting windings appears to be an expensive luxury, while the alternative of using steel, in addition to increasing the magnet cost by about 5-10%, makes the system much more bulky and less accessible. It might be preferable, therefore, not to have to provide shielding except for special purposes (e.g. rf cavities); a more detailed survey of this problem is needed.

Also under this heading the question of remanent fields can be mentioned. Although large in present superconducting coils, these are automatically reduced, along with the ac loss, by the use of finer superconducting filaments, and are expected eventually to be lower than in iron-cored magnets. This would mean that an accurate field distribution might be retained at lower fields than in conventional magnets, allowing the use of a lower energy (and cheaper) injector; in practice, however, it will not usually be possible to take advantage of this, since space charge limitations necessitate a high injection energy to achieve adequate beam intensity.

5. Power Supply

The usual assumption is that conventional power supply systems will be used, with the operating current similar to that of conventional synchrotron magnets, i.e. a few thousand amperes, and with the magnet ring subdivided as usual to reduce the excitation voltage to a few kilovolts. Lower currents would result in higher voltages or an unreasonable amount of subdivision of the circuit; higher currents would increase the complexity of the superconducting cable, but values in the 10 000 to 20 000 A region might well be considered. To reduce heat leak into the system, current leads need only be brought out to room temperature for connection to the supply, interconnections between magnets being kept at 4^{o}K.

With the high current density coils envisaged, any shorts or normal regions formed within the coil would result in local damage by overheating or voltage breakdown within a few milliseconds. Protection against this appears almost impossible, the usual methods available for dc magnets being unsuitable for pulsed operation. It will undoubtedly be possible, however, to confine the damage to a single magnet unit, so the best solution appears to be to design the coils with an adequate safety margin, and replace individual units in the event of an occasional failure.

Since, in contrast to conventional magnets, no provision has to be made for dc power requirements under "flat-top" conditions, and since there is negligible energy loss per cycle, the possibility can be envisaged of alternative power supply schemes utilizing energy storage in superconducting magnets. This is discussed in a separate paper.[4]

IV. SPECIFIC DESIGNS

Some exploratory studies have been made of possible schemes for the conversion or extension of the existing accelerators at the Rutherford Laboratory. Although of somewhat limited general interest, they nevertheless provide a useful guide to the typical parameters of superconducting magnet lattices.

It should be emphasized that no proposal of this nature is being considered at present; the calculations have been carried out simply to obtain a preliminary impression of what might be possible in the future.

Separated function lattices are used throughout, designed with the aid of a computer program and optimized for approximate minimum aperture.

1. Original Nimrod Conversion

Nimrod is a constant gradient proton synchrotron with a 15 MeV injector and an energy of 7 GeV (7.9 GeV/c momentum) at the normal 14 kG peak magnetic field.

The original suggestion was simply to replace the large aperture constant gradient magnet with a small aperture superconducting alternating gradient magnet which, at ~ 70 kG, would have about the same stored energy, ~ 40 MJ. This might allow the energy to be increased to ~ 40 GeV using all the existing facilities except the magnet and rf system. Alternatively, with a separate (larger radius) magnet tunnel and a mean field of 50-60 kG, an energy of 50 GeV might be achieved with the existing power supply.

Closer examination showed that this scheme would not satisfy the future necessity for a higher beam intensity, since, with the assumed aperture radius ~ 5 cm, the space charge limit would be $\leq 3 \times 10^{12}$ protons/pulse. Nearby, however, there is an operational 50 MeV linear accelerator, which could be used to replace the existing injector

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4. P.F. Smith, these Proceedings, p. 1002.

and increase the space charge limit to $\sim 10^{13}$ protons/pulse. Better still, of course, a completely new injector could be provided.

A typical computed lattice is shown in Fig. 3, assuming 60 kG in the bending magnets and peak fields of 60-70 kG in the quadrupoles. With an emittance of 150 μm·R (corresponding to $\sim 10^{13}$ protons at 50 MeV) the beam radius for this optimum lattice is about 4.1 cm, to which must be added perhaps 0.5 cm for closed orbit distortions. One long bending magnet is provided per period, which can be omitted to provide a straight section about 4 m long. Assuming only four straight sections and a maximum mean radius of 26 m, one obtains a particle energy of 30 GeV, which represents a more realistic upper limit of what might actually be achieved in the existing magnet hall.

2. Single Stage 180 GeV Machine

As a second exercise, the possibility of a larger magnet ring was considered, again using the 50 MeV injector. Guessing a lower limit of 100 G for the injection field, a peak field of 60 kG gives an energy of 180 GeV. Preliminary trials showed that of various quadruplet, triplet, and FODO lattices, the latter gave the smallest apertures for a given peak quadrupole field.

The parameters of one of the best lattices are summarized in Fig. 4. This time a more realistic allowance is made for the ratio of peak field/usable field in the quadrupoles, and the ratio mean radius/bending radius was fixed at 1.35 (or $\sim$ 1.6 with insertions). With these assumptions the beam radius is about 5 cm for an emittance of 75 μm·R, corresponding to only 5×10^{12} protons.

In this machine the required straight section length would probably be $\sim$ 15 m for extraction purposes, and so would have to be provided by matched insertions rather than by the omission of bending magnets.

3. Conversion of Nimrod to Booster

The beam intensity in the previous machine could be increased simply by increasing the beam aperture radius a ($a \propto \sqrt{N}$), but the figures in Section II show that the cost begins to increase rapidly for $a > 7$ cm. A less expensive solution is to provide a booster synchrotron at an intermediate energy.

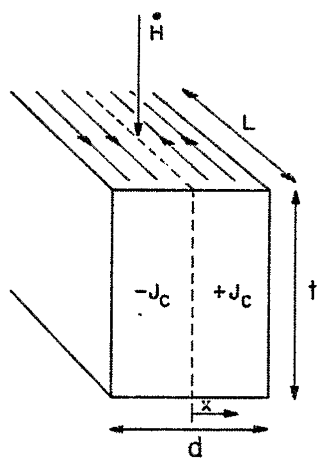
The third exercise, therefore, was to consider the conversion of Nimrod into a 5-25 GeV booster, followed by a high energy machine of very small aperture. In the 180 GeV ring, for example, the space charge limit would allow containment of 4×10^{13} protons in a beam radius of only $\sim$ 1 cm at 5 GeV injection, and $\sim$ 0.5 cm at 25 GeV injection. Since these numbers are now becoming smaller than the allowance necessary for coil former and thickness, the advantage in injecting at 25 GeV rather than 5 GeV is not very great. This immediately suggests the possibility of schemes in which there is an initial conversion to, say, 20-25 GeV, and the same magnet is subsequently used at $\sim$ 5 GeV as a conventional fast cycling booster for a higher energy machine. At 180 GeV, the radius ratio would be $\sim$ 6, so that $> 7 \times 10^{12}$ protons/pulse in the booster would be necessary to give 4×10^{13} protons in the main ring. Because of the reduced field, fast cycling at 3-4/sec should not result in any significant power supply or ac loss problems.

A variety of calculations along these lines are in progress. A typical 25 GeV lattice is shown in Fig. 5, requiring a beam radius of $\sim$ 3.8 cm for 10^{13} particles. In general, for a fixed machine radius of $\sim$ 20 m, the computed beam radii fit the expression $a = 3.3 \times 10^{-7}\, p^{0.4}\, N^{0.5}$, where N is the number of particles, for values of the momentum p between 5 and 30 (GeV/c).

For the main ring, if the lattice of Fig. 4 is used, the beam radius is now reduced to $\sim$ 1.1 cm for 4×10^{13} particles. The smaller aperture, however, allows the use of a higher field gradient so the lattice must be reoptimized; the result is shown in Fig. 5 and the new beam radius is $\sim$ 0.9 cm. Only a small gain results, therefore, from the increase in quadrupole strength (principally because the quadrupoles occupy only about 15% of the circumference) and the optima are, in any case, fairly flat. Much larger changes in beam radius can result, however, from a change in the type of lattice; triplet lattices, for example, appear to require apertures typically twice as great as FODO lattices.

Cost estimates have not yet been made for the above schemes, nor has any detailed consideration been given to rf requirements, injection and extraction efficiencies, etc.

Since the computed lattices cover a fairly wide range of energy and intensity, the magnet parameters are likely to be fairly typical of future requirements. It appears, therefore, that for a 60 kG peak field we can expect bending magnet lengths in the range 1 to 6 m, quadrupole lengths of 0.3 to 1.2 m, and aperture radii between 2 and 6 cm.



$$\text{POWER} = \int JE\, dV = \int_{-d/2}^{+d/2} J_c \times (\dot{H}x/10^8) \times Lt\, \delta x$$

$$= \dot{H} J_c \times Ltd \times d/4 \times 10^8$$

$$\dot{Q} = \dot{H} J_c V d/4 \times 10^8$$

Fig. 1. Basic ac loss calculation.

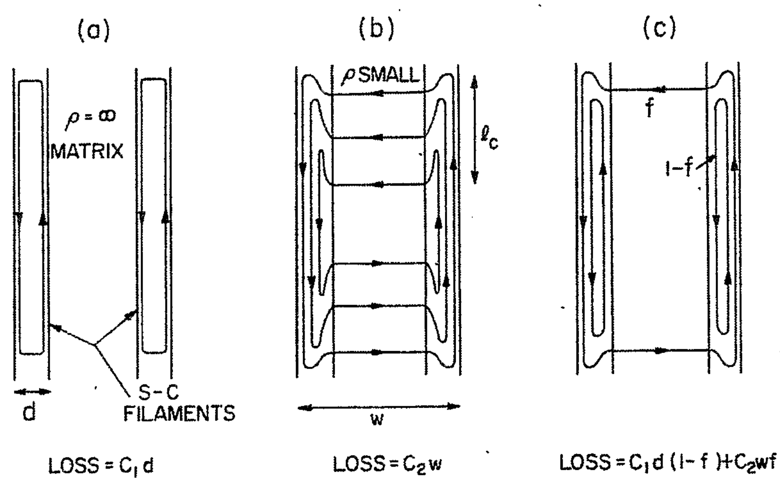
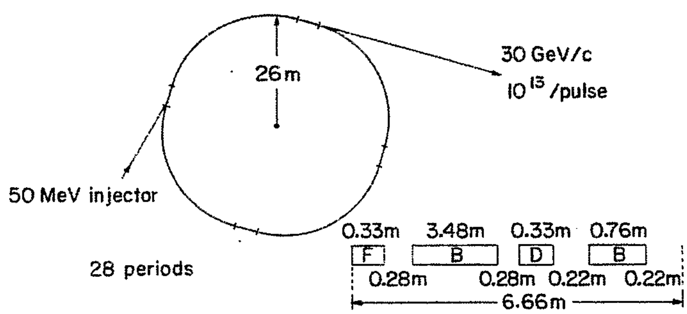


Fig. 2. Distribution of magnetization currents in a composite conductor.



(long bending magnet omitted in 4 periods to give 4 x 4 m straights)

bending field	60 kG
quadrupole field gradient	12 kG/cm
beam radius	4.1 cm
no. of betatron oscillations (Q)	4.75
transition energy/rest energy (γ_T)	4.5

Fig. 3. Typical lattice for Nimrod conversion.

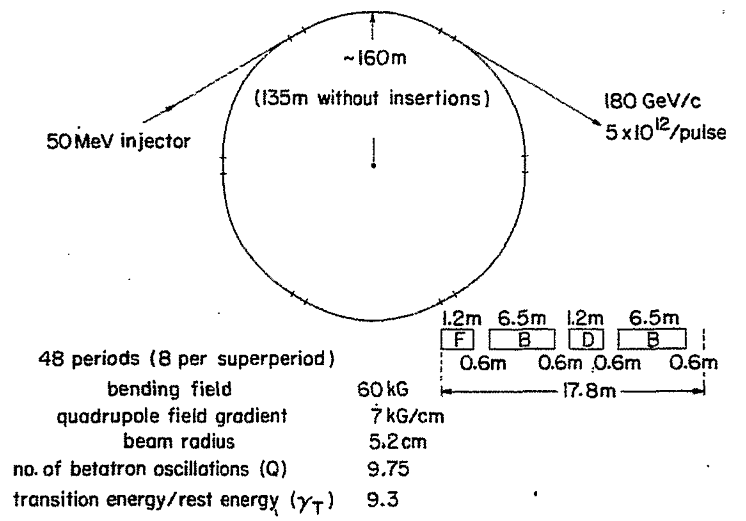


Fig. 4. Typical 180 GeV lattice with 50 MeV injection.

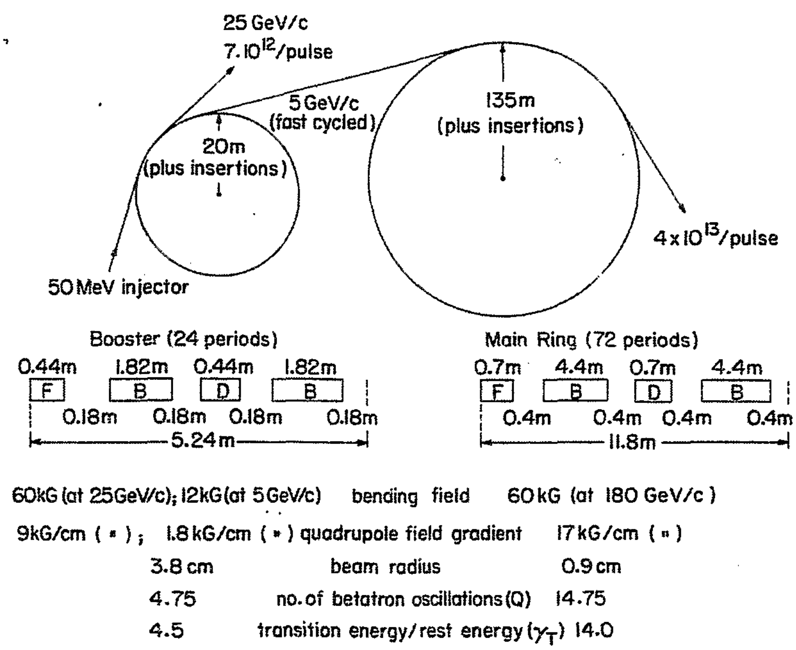


Fig. 5. Typical lattices for 180 GeV with 5 GeV booster.