# SUPERCONDUCTING FFAG ACCELERATORS*

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## I. INTRODUCTION

The possibility of superconducting high field magnets makes a nonpulsed accelerator like the FFAG (Fixed Field Alternating Gradient) seem attractive. It seems worthwhile to point out some characteristics of the FFAG which pertain to the possible use of this accelerator in the current state of the accelerator field.

One difficulty with the FFAG as it was conceived by the MURA group[1] was the large radial aperture required. This difficulty is to some extent ameliorated today for several reasons. One reason is that some accelerators being considered today have a low momentum ratio $p_f/p_i$, where $p_f$ and $p_i$ are the final and injection momenta of the particles. Some examples are:

a) A 200 MeV to 1 GeV booster, $p_f/p_i \simeq 3$.

b) A high energy second stage, 300 GeV to 1000 GeV, $p_f/p_i \simeq 3$.

The momentum swing of $p_f/p_i = 3$ in the above two examples is small enough to be achievable in an aperture of about 15 cm.

A second reason is that the extraction from one accelerator followed by injection into a second accelerator now seems more feasible. Experience with extraction over the past years has increased one's confidence in being able to extract and inject. Thus, rather than build one FFAG accelerator with a very large aperture, it appears possible to build an FFAG having several stages, each with a moderate aperture, where the beam is extracted from one stage and injected into the following stage. An example of such a multistage FFAG would be an accelerator to go from 30 GeV to 800 GeV in three stages, each stage having a $p_f/p_i = 3$. The three stages would be

$$30 \rightarrow 90,\ 90 \rightarrow 270,\ 270 \rightarrow 810\ .$$

One interesting result is that the radial aperture of each of the three stages would be the same. This result follows from the energy scaling properties of the FFAG and is discussed below. An aperture of about 15 cm in each stage may be satisfactory.

## II. CHOICE OF PARAMETERS

The horizontal aperture required for the accelerator depends on how large one can make the magnetic field gradient. However, the magnetic field gradient is limited by its effect on the stability limits for the betatron oscillations. The higher the gradient, the smaller the stability limits will be.

It is worth noting here that the FFAG is a high repetition rate machine, which means it can go through its acceleration cycle many times per second. A repetition

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1. MURA Proposal for a 10 GeV Accelerator, 1962.

rate of 30/sec to 60/sec seems reasonable.[1] Thus, the stability limits can be quite small for the three examples mentioned in the Introduction.

A stability limit of $A_r = 1$ cm has been assumed in this paper. This choice should be reconsidered for the particular accelerator being designed. Further considerations on the stability limit and repetition rate are given in Section III.

The manner in which the accelerator parameters vary with the particle energy or the accelerator radius, R, follows from a consideration of the stability limits. It is shown in the Appendix that the radial aperture is energy independent for a given momentum swing $p_f/p_i$ and is given by

$$\Delta R = 7.53 \frac{A_r}{f} \log (p_f/p_i) \quad , \tag{1}$$

when f is the azimuthal flutter in the magnetic field. We find $\Delta R = 8.26$ cm for $A_r = 1$ cm, $f = 1$ and $p_f/p_i = 3$. If we allow room for betatron oscillations, synchrotron oscillation and central orbit errors, assuming rather arbitrarily 1 cm for each, and allowing 0.3 cm for central orbit scalloping, we get $\Delta R = 15$ cm.

The various parameters of several accelerators having different final energies are given in Table I. The maximum field in the median plane was assumed to be 50 kG, and a circumference factor of 2 was assumed. The median plane magnetic field varies according to

$$H_z = B_o (r/r_o)^k \left[ 1 + f \cos \left( N\theta - P \log (r/r_o) \right) \right] . \tag{2}$$

Equation (2) indicates the meaning of the parameters k, P, N, f. The parameter P determines the spiral of the magnetic field. The azimuthal periodicity is given by $2\pi/N$. f is the flutter of the field in the radial direction. k determines how rapidly the field rises in the radial direction. f was assumed to be 1. The choice of parameters given in Table I was found using the results listed in the Appendix.

TABLE I. A table of parameters of various accelerators having a final energy $E_{max}$ ranging from 0.2 to 800 GeV. Each accelerator has a maximum median plane field of 50 kG, a circumference factor of 2 and a momentum swing of 3. $\Delta R$ is the good field radial aperture.

| | | | | | | |
|---|---|---|---|---|---|---|
| $E_{max}$ (GeV) | 0.2 | 1 | 30 | 90 | 270 | 800 |
| $E_{inj}$ (GeV) | 0.05 | 0.2 | 10 | 30 | 90 | 267 |
| R (m) | 0.86 | 0.26 | 41.26 | 121 | 361 | 1069 |
| N | 18 | 30 | 124 | 212 | 366 | 632 |
| $\nu_r$ | 3.6 | 6.2 | 24.8 | 42.4 | 73.2 | 126.4 |
| $\nu_z$ | 2.5 | 4.2 | 17.5 | 2.90 | 51.6 | 89.1 |
| k | 11.4 | 30 | 543 | 1614 | 4807 | 14 218 |
| P | 73 | 192.1 | 3509 | 10 334 | 30 779 | 91 038 |
| f | 1 | 1 | 1 | 1 | 1 | 1 |
| $\Delta R$ (cm) | 15 | 15 | 15 | 15 | 15 | 15 |

## III. MATCHING OF BEAM INTENSITY AND PHASE SPACE

At this time there is interest in using a superconducting FFAG in connection with existing conventional accelerators. This raises the question as to whether the relatively small radius of the superconducting accelerator will give rise to problems as regards space-charge limits and in the matching of the emittance and acceptance of the accelerators.

To illustrate the problems that arise, an example will be considered which is the use of a superconducting FFAG booster to raise the injection energy of the 200 MeV linac to 1 GeV for injection into the Brookhaven AGS. The following arguments and calculations are very rough and are meant only to be indicative of the kind of solutions available.

In this example there are three aspects to be considered. These are: 1) space-charge limitations, 2) injection into the booster from the 200 MeV linac, and 3) extraction from the booster into the AGS at 1 GeV.

### Space-Charge Limitations

The question here is whether the FFAG with a radius of 2.26 m and a stability limit of about 1 cm can contain enough charge to inject into the AGS at 1 GeV.

The incoherent space-charge limit is given very roughly by

$$N \cong \frac{a\ (a + b)}{R}\ \nu \cdot \Delta\nu\ \gamma\beta^2 \quad , \tag{3}$$

where a more conservative $\gamma\beta^2$ dependence is used instead of $\gamma^3\beta^2$. a and b are the horizontal and vertical apertures respectively. $\nu$ is the betatron oscillation frequency and $\Delta\nu$ is the allowable shift in $\nu$ due to space charge. R is the radius of the accelerator. $\gamma$ and $\beta$ are the particle energy $E/mc^2$, and the particle velocity v/c.

If we assume that the AGS at 200 MeV has the space-charge limit of N (AGS, 200 MeV) = $1.0 \times 10^{13}$ protons, then Eq. (3) indicates that at 1 GeV, N (AGS, 1 GeV) = $4.0 \times 10^{13}$ protons.

The injection space-charge limit of the FFAG may be found by comparison with the AGS at 200 MeV and using Eq. (3). The FFAG space-charge limit when compared with the AGS space-charge limit is larger by the radius factor 120/2.26 = 53, is smaller by the aperture factor 5(5 + 2.5)/1(1.0 + 1.0) = 18.7, and is smaller by the $\nu$ factor 8.25/4.2 = 2.08. Thus we find

$$N\ (\text{FFAG, 200 MeV}) = 1.36\ N\ (\text{AGS, 200 MeV}) \quad ,$$

$$N\ (\text{FFAG, 200 MeV}) = 1.36 \times 10^{13}\ \text{protons} \quad .$$

The $4 \times 10^{13}$ protons required by the AGS at 1 GeV can be obtained if the repetition rate of the FFAG is larger than 3/sec.

### Injection into the FFAG

The 200 MeV linac has difficulty injecting enough protons into the FFAG because of the small time/turn due to the small radius. The time/turn at 200 MeV for the FFAG is $\tau = 0.084$ µsec. If the linac is assumed to deliver 50 mA, then it can inject $2.5 \times 10^{10}$ protons per turn.

The number of turns that can be accepted is determined by the acceptance of the FFAG, which we will assume given by the rough formula

$$\text{Acceptance} \cong \frac{\nu a^2}{R} \quad . \qquad (4)$$

Using a = 1 cm, $\nu$ = 6.2, we find the FFAG horizontal acceptance at 200 MeV,

$$\text{AC (FFAG, 200 MeV)} = \pi\, 27.4 \times 10^{-3}\ \text{cm}\cdot\text{rad} \quad .$$

If we assume the emittance of the linac is $\pi\, 1 \times 10^{-3}$ cm·rad, then we have room for 27 turns, or $6.75 \times 10^{11}$ protons can be injected in 27 turns.

Thus in order to be able to inject the $4 \times 10^{13}$ protons required by the AGS at 1 GeV, it is necessary for the FFAG to have a repetition rate of 60/sec.

An alternative to the repetition rate of 60/sec is to increase the stability limit to perhaps 1.4 cm, which would allow a repetition rate of 30/sec.

One may note that at the present time the Brookhaven 200 MeV linac is designed with a repetition rate of 10/sec.

Extraction from the FFAG into the AGS at 1 GeV

First let us compare the emittance of the FFAG at 1 GeV with the acceptance of the AGS at 1 GeV.

Because the momentum swing of the FFAG is $p_f/p_i$ = 3, the emittance at 1 GeV is one-third the acceptance at 200 MeV or the emittance is

$$\text{EM (FFAG, 1 GeV)} = \pi\, 9.1 \times 10^{-3}\ \text{cm}\cdot\text{rad} \quad .$$

The acceptance of the AGS at 1 GeV can be compared with the emittance of the FFAG at 1 GeV using Eq. (3). It is larger by the aperture factor $(5\sqrt{3})^2$ = 75, smaller by the radius factor 120/2.26 = 53, and larger by the $\nu$ factor 9/6 = 1.5. We find that

$$\text{AC (AGS, 1 GeV)} = 2.1 \times \text{EM (FFAG, 1 GeV)} \quad .$$

One may note that the matching of the phase spaces is due to the cancellation of the radius factor by the aperture factor, which seems to require the stability limit of the FFAG to be relatively small or the emittance of the FFAG will become too large for injection into the AGS.

The transfer of the charge between the FFAG and the AGS is complicated by the radius factor of 53. A distance of $2\pi \times 2.26$ m in the AGS can only contain 1/53 of the space-charge limit of $4 \times 10^{13}$ protons.

One possible method is to inject 60 times/sec into the FFAG and to stack the 60 pulses at 1 GeV. The stacked beam, containing roughly $4 \times 10^{13}$ protons, can be extracted over 53 turns in the FFAG and injected over the entire circumference of the AGS. Needless to say, there are many problems to be solved in such a scheme.

A second method would be to extract each of the 60 pulses in the FFAG after it is accelerated to 1 GeV and inject it over $2\pi \times 2.26$ m in the AGS. If the transfer of charge is asynchronous, the process is likely to be less efficient.

In conclusion, it would seem that using a high repetition rate and a small aperture or stability limit with the small radius superconducting accelerator can produce a satisfactory match with a conventional accelerator. The above discussion should apply also to a superconducting pulsed AGS.

## IV. A 200 MeV TO 1 GeV BOOSTER

A better idea of the magnetic field configuration in a superconducting FFAG may be obtained by considering a particular accelerator in more detail. For this purpose, let us consider a 200 MeV to 1 GeV FFAG booster which might be used in conjunction with the Brookhaven AGS.

A possible set of parameters for this booster are the following:

| | |
|---|---|
| Inside radius, R | 250 cm |
| Good field aperture, $\Delta R$ | 17 cm |
| Periodicity, N | 24 |
| $\nu_r$ | 4.8 |
| $\nu_z$ | 3.3 |
| Radial gradient, k | 20.5 |
| Spiral gradient, P | 131 |
| Spiral angle, $\alpha$ | $10^o$ |
| Flutter, f | 1 |
| Maximum field | 50 kG |
| Injection field | 13 kG |
| Horizontal stability limit, $A_x$ | ± 1.62 cm |
| Vertical stability limit, $A_y$ | ± 0.46 cm |
| Vertical aperture | ± 3 cm |

The above parameters are somewhat different from those given in Table I, and they are a little more on the conservative side. The stability limits $A_x$, $A_y$ are given at the azimuth where the horizontal $\beta$ has a maximum.

### Median Plane Magnetic Field

The magnetic field in the median plane is indicated in Fig. 1. The crosshatched areas indicate the regions of high field, the blank areas the regions of zero field. The region which will be crossed by protons is bounded by the dashed circles.

A possible magnet arrangement is shown in Fig. 1. In this arrangement the magnet edges are along radii to the center of the machine, and within each magnet the spirals may possibly be replaced by straight lines.

An alternate arrangement for a small accelerator is to use long spiral magnets whose shapes would be similar to the crosshatched areas in Fig. 1.

## V. POSSIBLE PROBLEMS OF THE SUPERCONDUCTING FFAG

One aspect of the FFAG that may prove a serious problem has to do with the ratio of the maximum magnetic field at the current-carrying coils to the magnetic field on the median plane. If this ratio is too large, then the superconducting FFAG loses its advantage as the field at the coils is limited to some value of the order of 100 kG. This field ratio tends to get larger when the field has a rapid radial variation as is required in the FFAG.

Some other problems include that of determining the current distribution to obtain the desired median plane magnetic field, and the problem of putting radial straight sections in the FFAG. Some further considerations of particular magnets and machine design are reported elsewhere.[2]


## ACKNOWLEDGEMENTS

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## APPENDIX

Some handy formulas are available for estimating the parameters of the FFAG.

The stability limits at some optimized operating point are related to the parameters by[3]

$$A_r/R = 3.74\ f/N^2$$
$$A_z/R = 5.12\ f/N^2 \tag{A.1}$$

$$k/N^2 = 0.03559$$
$$fP/N^2 = 0.2279 \tag{A.2}$$

$$\nu_r/N = 0.2007$$
$$\nu_z/N = 0.1410 \quad . \tag{A.3}$$

The radial aperture is given by

$$\Delta R = \frac{R}{k+1} \log (p_f/p_i) \quad , \tag{A.4}$$

where $p_f$ and $p_i$ are the final and initial momenta.

Rough formulas for $\nu_r$ and $\nu_z$ are

$$\nu_r^2 = k + 1$$
$$\nu_z^2 = \frac{f^2 p^2}{N^2} - k + \frac{f^2}{2} \quad . \tag{A.5}$$

For a given operating point $\nu_r/N$ and $\nu_z/N$, the stability limits are proportional to $R/N^2$, where $2\pi/N$ is the periodicity of the accelerator. If one assumes that each

2. P.G. Kruger and J.N. Snyder, University of Illinois Internal Report, Sept. 1966.

3. G. Parzen and P. Morton, Rev. Sci. Instr. 34, 1323 (1963).

stage of the accelerator is to have the same stability limits, it follows that the periodicity parameter N varies with the radius according to

$$N \sim \sqrt{R} \quad . \tag{A.6}$$

Since for an accelerator with a given operating point $\nu_r/N$, $\nu_z/N$, one must have $k/N^2$ = constant, where the field goes like $r^k$; one finds that the field gradient parameter k varies with radius according to

$$k \simeq R \quad . \tag{A.7}$$

The radial aperture, $\Delta R$, of the accelerator is given by Eq. (A.4).

Thus, for a given momentum swing $p_f/p_i$, the radial aperture varies like $R/(k + 1)$, since $k \simeq R$ is independent of R for large k. This establishes the result, mentioned in Section I, that the radial aperture is the same for each stage of the accelerator. Combining Eqs. (A.1), (A.2) and (A.4) gives the result for the radial aperture

$$\Delta R = 7.53 \frac{A_r}{f} \log (p_f/p_i) \quad . \tag{A.8}$$

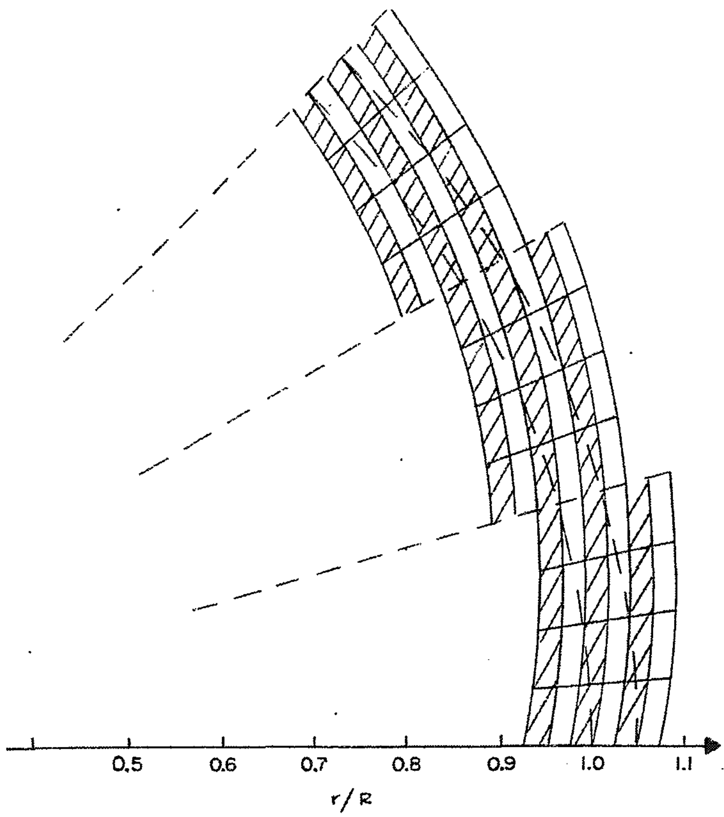


Fig. 1. The magnetic field in the median plane of an FFAG accelerator. Crosshatched areas indicate regions of high field. The period $2\pi/N$ is $15^o$ and the accelerator radius is 2.5 m.