# CALCULATIONS CONCERNING SUPERCONDUCTING ACCELERATORS 

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## INTRODUCTION

Part I of this paper considers the number and positioning of $\mathrm{Nb}_{3} \mathrm{Sn}$ superconducting ribbons to produce the magnetic field necessary for an FFAG (fixed-field alternatinggradient) separated-sector spiral-ridge accelerator. It also considers the necessary flux biasing so that the field between peaks, as the radius increases, does not become negative. The net field (biased field) for several values of $\theta$ (the azimuthal angle with respect to the center of the accelerator) is given. The field index, $k$, is found to be satisfactorily constant from $R=239 \mathrm{~cm}$ to $R=261 \mathrm{~cm}$. This means that a model FFAG accelerator built to these specifications should be able to accept 50 MeV protons which are injected and accelerate them to about 1.2 GeV . It would be an interesting model to build with the $\mathrm{Nb}_{3} \mathrm{Sn}$ now available:

Part II considers a separated-function pulsed superconducting accelerator for 30 to 300 GeV . The necessary magnetic dipoles and quadrupoles can be built from $\mathrm{Nb}_{3} \mathrm{Sn}$ now available. One disadvantage of this type of accelerator is that the pulsing produces heating which must be removed by the refrigerating system and this is costly. But some members of this Sumer Study believe that the cost will be reduced in the future so that it is not a serious inhibition to this type of accelerator.

A second disadvantage is that the pulsing demands a motor generator power source (or some similar equipment) so that the magnet may change the magnetic field from zero to a maximum magnetic field every 2 or 3 sec . Such capital equipment is costly and the operating cost for the many megawatts of power which are necessary to operate the magnets is a major item. Otherwise, these are interesting accelerators.

Part III concerns some parameters and properties of a superconducting FFAG accelerator for 30 to 300 GeV . Such an accelerator is feasible now with the $\mathrm{Nb}_{3} \mathrm{Sn}$ available and has the advantage that there is no heating owing to pulsing and thus no large expense is necessary for the refrigerating equipment. Secondly, no large and costly motor generator sets to run the magnet are necessary. It does have a small disadvantage owing to a circumference factor of 2 whereas the circumference factor for the pulsed accelerator is about 1.3 or 1.4 .

Part IV compares, on a relative basis, the estimated costs of some components of the pulsed superconducting accelerators with an FFAG superconducting accelerator. It appears that at the present time the FFAG accelerator is much less expensive than the pulsed accelerator. However if, in the future, the heating owing to pulsing can be reduced by a factor of 5 and if the cost of refrigerating equipment were to be reduced by a factor of 4 (a total factor of cost reduction of 20 ) then the estimated cost of these critical items would be about the same for the two systems. At that time the remaining disadvantage of the pulsed accelerator would be the large power bill in the operating cost.

## I. THE BROOKHAVEN FFAG MODEL

Parzen has discussed superconducting FFAG accelerators at this Summer Study and elsewhere. 1 Consequently, and in accordance with those ideas, a model of an FFAG accelerator has been under discussion at Brookhaven National Laboratory for some time. It is a separated-sector spiral-ridge accelerator and consists of 24 sectors ( $N=24$ ), each with eight subsections as shown in Fig. 1. Additional details of a spiral sector are given in Fig. 2 where, however, the number of turns per each subsection and the radii of the semicircular ends of the subsections are different from those used in the calculations described below.

While the general features of the Brookhaven model as given by Parzen and as drawn by D. Jacobus are maintained in these calculations, it is well known from the calculations done for the $M 4-24 Q$ quadrupole ${ }^{2}$ that it is necessary to make detailed magnetic field calculations using the actual currents in the ( $\mathrm{Nb}_{3} \mathrm{Sn}$ ) ribbon, if one is to have a good idea of the number of turns of ribbon necessary in the subsections, to produce a given magnetic field and a given field gradient (i.e., a specified field index, $k$ ). Accordingly, Part $I$ of this discussion deals with the magnetic field calculations for the model and the coil parameters deduced therefrom.

Preliminary field calculations ${ }^{3}$ for this model include a description of the method by which the magnetic field is calculated in the median plane of the accelerator. (It is perpendicular to the major axis of the ellipse in Fig. 5.) In these preliminary calculations all subsections were rectangular and had their axes along the same straight line. In the present calculations the various subsections of the spiral have axes which are tangent to the real spiral and straight sides parallel to the axes. The ends of the subsections approximate a semicircle with four tangent straight sections. This geometry is shown in Fig. 3 and represents the geometry used in these calculations. The approximations are thought to be a reasonably good approximation to a real spiral FFAG ridge. Table I gives some physical data for one spiral and such a spiral is repeated every $15^{\circ}$ around the accelerator.

TABLE I
Pertinent spiral coil data which with the data
in Fig. 3 specify one separated FFAG ridge

| Subcoil <br> number | Number of turns <br> per subsection | Length between <br> subsection <br> centers <br> $(\mathrm{cm})$ | Inside radius <br> of end turns <br> of subsection <br> $(\mathrm{cm})$ |
| :--- | :---: | :---: | :---: |
|  | 188 |  |  |
| 1 | 140 | 21.2725 | 1.763 |
| 2 | 100 | 16.8275 | 2.024 |
| 3 | 79 | 16.8275 | 2.237 |
| 4 | 79 | 16.8275 | 2.413 |
| 5 | 60 | 16.8275 | 2.555 |
| 6 | 44 | 16.8275 | 2.674 |
| 7 | 32 | 16.8275 | 2.770 |
| 8 |  | 20.9550 | 2.857 |

1. G. Parzen, Brookhaven National Laboratory, Accelerator Dept. Report AADD-138 (1967); G. Parzen and P. Morton, Rev. Sci. Instr. 34, 1323 (1963).
2. P.G. Kruger and J.N. Snyder, in Proc. 6th Intern. Conf. High Energy Accelerators, Cambridge, 1967, p. 391.
3. P.G. Kruger and J.N. Snyder, University of Illinois Report (1968).

Other pertinent data are:

1) The current in the ribbon is assumed to be 2000 A .
2) The ribbon is $\frac{1}{2}$ in. wide $\mathrm{Nb}_{3} \mathrm{Sn}_{\mathrm{S}}, 0.00625$ in. thick.
3) For the calculations the current is assumed to be in eight wires uniformly spaced in the ribbon rather than in a uniform current sheet.

The magnetic field resulting from this current distribution has been calculated for a system of three spirals and for a system of five spirals but not for 24 spirals which make a complete accelerator. There is no significant difference in the results for the two cases and thus all data below are for a system of three spirals. The field has been calculated along $R$ from $R=230 \mathrm{~cm}$ to 270 cm in steps of $\Delta R=1 \mathrm{~cm}$ and at various angles of $\theta$ where $\theta=0$ is the angular coordinate for the small end of one spiral which then ends at $\theta=32.872^{\circ}$.

Figure 4 shows the results of the calculations for $\theta=30.5^{\circ}$. It is noted in Fig. $4 a$ that as $R$ increases the field becomes negative between the peaks. This is undesirable and may be eliminated by flux biasing from current strips on an elliptical coil whose cross section is shown in Fig. 5. The necessary turns distribution for such flux biasing ${ }^{3}$ is given in Table $I I$ and the calculated net biasing field for the above case is given in Fig. 4b. Figure 4 c gives the net flux-biased field, i.e., the sum of the fields in Figs. $4 a$ and $4 b$.

TABLE II
Necessary Turns Distribution for Flux Biasing
(Note the $\pm$ signs denote the relative direction of the current in the strip.)


In Fig. 5 there is shown a schematic cross section of the vacuum chamber and the coil structure along a radius (the major axis of the ellipse). The spiral coils of Figs. 1 and 2 are represented by the open boxes above and below the major axis of the ellipse a distance $Z$ (here $Z=1.1 \mathrm{in}$. $=2.8 \mathrm{~cm}$ ). The elliptical structure encloses the whole spiral-ridge-field coils and also supports the biasing field coils listed in Table II. The turns are placed in the angular sections shown in the outer ellipse of. Fig. 5 and which are $\Delta \theta=2.5^{\circ}$ out to $\theta=10^{\circ}$ and then $\Delta \theta=10^{\circ}$ out to $90^{\circ}$. The turns wound in the $\Delta \theta$ angle intervals to the left of the posts would have the current in one direction whereas to the right it would be in the opposite direction.

Figure 6 gives the calculated magnetic field (biased) as a function of the radius for four values of $\theta$.

From these it is possible to construct Fig. 7 which shows the peak value of the magnetic field at its corresponding radius. From these data it is possible to calculate an average value of $k$ (the field index). From $R=239 \mathrm{~cm}$ to $\mathrm{R}=260 \mathrm{~cm}$, $k=20.44$ ( +0.27 or - 0.36) . It may be possible to improve further the turn distribution so that $k$ is even more constant. Also by a slight adjustment in the turn distribution $k$ may be reduced in magnitude so that $v_{z}$ (Table III) is larger than 3.5 .

Other pertinent parameters for the model are calculated ${ }^{4}$ in the usual way and are given in Table III. These appear to represent an interesting FFAG model accelerator which could be built with the superconducting $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbon now available.

TABLE III

## Pertinent Model Parameters

|  | Initial | Final |
| :---: | :---: | :---: |
| Energy (MeV) | 50 | 1230 |
| BR (kG.cm) | $1.03 \times 10^{3}$ | $6.52 \times 10^{3}$ |
| R accelerator (in.) | 94.1 | 102.7 |
| R accelerator (cm) | 239 | 260.7 |
| B peak accelerator (kG) | 8.62 | 50 |
| $B$ average accelerator (kG) | 4.31 | 25 |
| $\beta$ | 0.31406 | 0.9 |
| Rotational frequency ( $\sec ^{-1}$ ) | $6.28 \times 10^{6}$ | $16.5 \times 10^{6}$ |
| $v_{x}=\sqrt{k+1}$ | 4.63 |  |
| $v_{z}^{2}=-k+f^{2} \tan ^{2} E+\left(f^{2} / 2\right)$ | 3.50 |  |
| $\mathrm{f}=1$ (flutter) |  | - |
| 5 (degrees) | 80.11 |  |

4. K.R. Symon, D.W. Kerst, L.W. Jones, L.J. Laslett, and K.M. Terwilliger, Phys. Rev. 103, 1837 (1956).

## II. NOTES, COMMENTS AND CALCULATIONS CONGERNING A PULSED SUPERCONDUCTTNG ACCELERATOR

## 1. Introduction

Because of the success of the MURA ${ }^{5}$ separated.function storage ring and the design of the Weston 200 GeV accelerator as a separated function accelerator, there is much interest in the separated function superconducting accelerators. Furthermore, it is now well known that one can build superconducting quadrupoles ${ }^{2}$ and dipoles almost any size which one may want and of elliptic or circular cross section. These make.idedi. components for a separated function accelerator provided that the losses in the superconductor owing to pulsing are not so large as to obviate the other advantages of the superconductor. These general ideas are considered in these notes.

## 2. Components for a 30 to 300 GeV Accelerator

Dipoles. Gonsideration will be given to dipoles with a circular cross section and a length of about 6 m . This length is arbitraxily chosen and seems like a reasonable length from an engineering standpoint: for a magnetic field of 60 kG and for 300 GeV protons where $\mathrm{H} \rho=1.0038 \times 10^{6} \mathrm{kG} \cdot \mathrm{cm}, \rho=166.7 \mathrm{~m}$ and thus there would be 175 six-meter-long magnetic dipoles in the circumference of the accelerator.

The diameter of the circular cross section of the dipole is specified by considering the sagitta for a chord of 6 m in length and a radius of 167 m . This calculates to be 2.7 cm . To this must be added about $\pm 2 \mathrm{~cm}$ for betatron oscillations and some small amount (say 2.3 cm ) for design clearance, in which liquid helium may flow, and. wall thickness of the chamber. Thus the inside diameter of the magnetic field is taken to be 9.0 cm or about 3.5 in .

Accordingly, under these conditions and assumptions, the total stored energy in. . the magnetic field will be about $1.8 \times 10^{5} \mathrm{~J} / \mathrm{m}$ so that

$$
E_{\text {total, stored }}=1.9 \times 10^{8} \mathrm{~J}
$$

The necessary turn distribution of 0.5 in. $\mathrm{Nb}_{3} \mathrm{Sn}$ on a 3.5 in. diameter dipole can be estimated from the data in an earlier report ${ }^{6}$ wherein a 3 in. diameter dipole (M3-15D) was considered. From these data it is expected that the 60 kG field would be attained (conservatively) from the same curns distribution as that in Table VI of that report and that the field would be flat to about $\pm 0.1 \%$ over the requisite diameter.

The total length of $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbons which are necessary is a minimum of $1.22 \times 10^{6} \mathrm{~m}$ for the dipoles.

Quadrupoles. The $M 4-24 Q$ quadrupoles ${ }^{2}$ already built and tested at Brookhaven surely are more than adequate for this separated function accelerator. Since the focal lengths of a pair of quadrupoles are inversely proportional to the square of the field gradient and since the gradient in $M 4-24 Q$ is about $16 \mathrm{~kg} / \mathrm{in}$. a quadrupole, M4-160 (i.e., 16 in. long instead of 24 in. long) will suffice. M4-240 has 1280 turns which at 50 cm per turn (conservative) give 640 m of Nb 3 Sn ribbon necessary for each quadrupole.
5. E.M. Rowe and F.E. Mills, University of Wisconsin Physical Sciences Laboratory, private communication.
6. P.G. Kruger and J.N. Snyder, University of Illinois Report (1967).

It is estimated that for the M3.5-240D dipoles one needs 318 quadrupoles and consequently $2.04 \times 10^{5} \mathrm{~m}$ of $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbon.

About $10^{7} \mathrm{~J}$ will be the total stored energy in the magnetic field of the quadrupoles. These data are summarized in Table IV.

TABLE IV

## Summary of Some Parameters for a Pulsed Superconducting

 Separated Function AcceleratorMaximum proton energy (GeV)
300
$\mathrm{H} \rho$ ( $\mathrm{kG} \cdot \mathrm{cm}$ )
Average dipole field (kG)$1.0038 \times 10^{6}$
Length of dipole magnet sections (m)60
Number of dipole magnet sections ..... 175
$\rho$, protons path radius in the magnetic field (m) ..... 166
$2 \pi \rho$, magnetic field path in accelerator (m) ..... 1047
Diameter of dipole cross section (in.) ..... 3.5
Length of 0.5 in . ribbon $\mathrm{Nb}_{3} \mathrm{Sn}$ for dipoles (m)
$1.22 \times 10^{6}$
Length of 0.5 in. ribbon $\mathrm{Nb}_{3} \mathrm{Sn}$ for quadrupoles (m)
$0.2 \times 10^{6}$
Total length of $\mathrm{Nb}_{3} \mathrm{Sn}(\mathrm{m})$
Total stored energy dipoles and quadrupoles (J)
$1.45 \times 10^{6}$$2.0 \times 10^{8}$
III. PARAMETERS AND PROPERTIES OF A SUPERCONDUCTING FFAG ACCELERATOR • 30 TO 300 GeV PROTONS

## 1. Introduction

Many of the ideas expressed or used in Part III of this paper are based on those developed by Parzen. 1 These, together with the superconducting properties of Nb 3 Sn , make it feasible and reasonable to consider the construction of superconducting accelerators in the region of 30 to 300 GeV as well as in the region of 300 GeV to 3 TeV . Some of the notations used in Table $V$, where some, parameters for a 30 GeV to 300 GeV accelerator are listed, are those of Symon et al. ${ }^{4}$

As mentioned by Parzen ${ }^{1}$ two of the most important reasons for again considering FFAG structures for accelerators are:
a) $\mathrm{Nb}_{3} \mathrm{Sn}$ makes large constant magnetic fields practical.
b) The small $\Delta R$ and large $k$ now realizable make the structure economical and practical.

However, rather than 1 imit $\Delta R$ to about 15 cm and $\mathrm{Pf}_{\mathrm{f}} / \mathrm{pi}_{\mathrm{i}}$ to about ${ }^{1} 3$ which would necessitate many extractions and injections from ring to ring of successive accelerators, it will be considered here that $p_{f} / p_{i} \cong 10$ and that $\Delta R \cong 60 \mathrm{~cm}$ (see Table V). This leads to a structure which has a reasonable engineering size and saves on extraction and injection. The circumference factor will be 2 and the flutter 1 . Moreover,
the inside clearance between coils is assumed to be $2 Z=2.2$ in. so that $Z=1.1 \mathrm{in} .=2.8 \mathrm{~cm}=A_{T} .{ }^{1}$

## 2. Determination of Constants (see Table V)

a) Choice of the magnetic field: The maximum magnetic field at the median plane of the FFAG structure is chosen to be 50 kG . This makes the estimated field at an $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbon in a coil about 87 kG and according to the RCA superconductivity properties of the 0.5 in. wide ribbon, the critical current at this field is 1525 A.
b) The field index $k$ : This is given by the usual expression $\left(B / B_{o}\right)=\left(R / R_{0}\right)^{k}$ which yields $k=1515$.
c) The radial betatron frequency: This is given by $\sqrt{k+1}=38.9$.
d) Now, the vertical betatron frequency is given by ${ }^{4}$ $\nu_{z}^{2}=-k+f^{2} \tan ^{2} \xi+f^{2} / 2$ and this is satisfied by: $\nu_{z}=26.8$, $\tan \xi=47.35, \xi=88^{\circ} 47^{\prime}$. It is considered feasible to construct a spiral sector with $\xi=88^{\circ} 4^{\prime}$.
e) Next $\Delta \mathrm{R}$ can be calculated from Parzen, ${ }^{1} \mathrm{Eq}$. (A.4). This gives the value listed in Table $V$.
f) Finally, the radial separation between points of maximum magnetic field is calculated to be $\Delta x=R \lambda=23.04 \mathrm{~cm}=9.08 \mathrm{in}$.
g) The lengths of a spiral sector are 28.16 m .

TABLE V
Some Parameters for a 30 to 300 GeV FFAG Accelerator

|  | Initial | Final |
| :---: | :---: | :---: |
| Energy (GeV) | 30 | 300 |
| HR ( $\mathrm{kG} \cdot \mathrm{cm}$ ) | $1.0315 \times 10^{5}$ | $1.0038 \times 10^{6}$ |
| H(av) (kG) | 2.58262 | $\cdot 25$ |
| H (max) (kG) | 5.16524 | 50 |
| R (cm) | $3.994 \times 10^{4}$ | $4 \times 10^{4}$ |
| R (m) | 399.4 | 400 |
| $\Delta \mathrm{R}$ (cm) (assumed) | 60 |  |
| N | 230 |  |
| k | 1515 |  |
| $\mathrm{p}=(1 / \mathrm{w})=\mathrm{N} \tan \underline{\Sigma}=(2 \pi / \lambda)$ | $1.09 \times 10^{4}$ |  |
| $\tan \xi$ | 47.35 |  |
| $\xi$ | $88^{\circ} 47^{\prime}$ |  |
| $\lambda=(2 \pi / \mathrm{p})$ | $5.76 \times 10^{-4}$ |  |
| $\Delta \mathrm{r}=[\mathrm{R} \lambda \equiv 9.08 \mathrm{in}].(\mathrm{cm})$ | 23.04 |  |
| $\nu_{x}$ | 38.9 |  |
| $\nu_{z}$ | 26.8 |  |

## 3. Some Details of Coil Construction

Based on the model design (Part $I$ ) and the field calculations for the model, it is possible to extrapolate those data to give preliminary constants for the spiral sector coils for the 30 GeV to 300 GeV FFAG accelerator. Accordingly, it will be considered that the spiral coils are 28.16 m long, have 12 subsections, that each subsection is 2.34 m in length and that each spiral sector is separated from its neighbor by 10.92 m along the circumference. These details are depicted in Fig. 8.

## IV. A COST COMPARISON OF SOME COMPONENTS OF SUPERCONDUCTING ACCELERATORS

The purpose of this part of the discussion is to present a comparison of the estimates of the costs of certain critical items for a pulsed and for an FFAG accelerator. It should be clearly understood that these are not estimated costs for an accelerator and that they are not accurately estimated costs but that the relative costs should be reasonably realistic.

For example, the cost of Nb 3 Sn for the two different types of accelerator is larger for the FFAG than for the pulsed accelerator. This is owing to the fact that the vacuum chamber (i.e., the magnetic field volume) is larger for the FFAG than for the pulsed machine. On the other hand, the costs for refrigeration to remove the heat produced by pulsing are large in the pulsed machine whereas they are nil for the FFAG machine. Correspondingly, the cost for a motor generator set or other similar equipment to provide the pulsed energy at the proper time interval (a few seconds) for the pulsed accelerator is large whereas for the FFAG accelerator it is negligible. Other costs as presented in Table VI are somewhat in favor of the pulsed accelerator, owing to the larger circumference factor for the FFAG machine.

The cost of the superconducting $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbon is based on an estimated cost for the future and for a large quantity of the superconductor. It seems like a realistic price. (See notes on cost items.) In any case the amount of the needed superconducting ribbon is reasonably accurately estimated so that even if the absolute cost were in error the relative cost would be quite good.

The largest uncertainty in these cost estimates is that owing to the refrigeration necessary to remove the heat developed by pulsing the superconductor. This cost differs by about a factor of 20 from the present to the optimistic future cost which some people envision. At the present time the FFAG accelerator surely has a large cost advantage and even in the future would break even unless the present losses owing to pulsing are greatly reduced and concurrently the cost of refrigerating equipment is greatly reduced.

For larger accelerators the cost should scale approximately with the radius.
No rf costs are listed in Table VI.
Another disadvantage to the pulsed accelerator is that during operation many megawatts of power will be consumed by the magnetic structure and will need to be purchased. For the FFAG accelerator this operating cost is negligible.

Other possible disadvantages and advantages:
a) The pulsing of present accelerators has been responsible for some mechanical failure, problems; and difficulties. At the larger magnetic fields envisioned here the corresponding magnetic pressure may be 15 to 20 times larger than in present
machines since the pressure is proportional to $B^{2}$. Thus, one should not be surprised if it should be necessary to design to only $40 \%$ of the tensile strength of materials for the pulsed machine compared to $80 \%$ for the FFAG accelerator. It might also be that material fracture would result from pulsing at such high fields.
b) Possibly one might get larger beam currents. from an FFAG accelerator than from a pulsed machine if one used the beam stacking techniques developed by Mura but, of course, one cannot extract more beam than is injected. Also, the power supply for the rf system will be relatively more costly for larger beam currents.

TABLE VI
A Cost Comparison for some Critical Items for Superconducting Accelerators with an Initial Energy of 30 GeV to a Final Energy of 300 GeV
(Estimated cost in millions of dollars)

1. Cost of $\mathrm{Nb}_{3} \mathrm{Sn}$ for magnets
a. Unbiased magnetic field coil
b. Biasing magnetic field coil
c. For dipoles
d. For quadrupoles
2. Cryogenic costs: refrigeration
a. Owing to ordinary heat leaks
b. Owing to pulsing the stored energy in the dipoles and quadrupoles i. At present prices ii. At optimistic future prices
3. Magnet power supply
4. Housing for magnet power supply and primary and secondary power lines
5. Vacuum system
6. Ring building for the magnet
7. Miscellaneous engineering costs

| Pulsed Cost |  | FFAG Cost |
| :---: | :---: | :---: |
| Present | Future Optimistic | Present |
|  |  | 5.5 4.2 |
| 3.7 | Same |  |
| 2.9 | Same | 5.0 |
| 100 |  | . |
| 30 | 15 | 0.02 |
| 2.0 | Same | Nil |
| 0.55 | Same | 1.3 |
| 4.1 | Same | 7.5 |
| 6.8 | Same | 12.5 |

Total relative estimated cost for these critical items

$$
150.7
$$

40
36

## Notes on Cost Items

1a. From Part III: (Eor spiral sectors)
(4000 m per sector) 2 (230 sectors) $\$ 3.00 / \mathrm{m}$ (Ref. 7 ) $=\$ 5.52 \times 10^{6}$
1b. From Part III: (for flux biasing)
(2Tp) [552 turns] $\$ 3.00 / \mathrm{m}$
$=\$ 4.16 \times 10^{6}$

1c. From Part II:
$\left(1.22 \times 10^{6}\right.$
m) $\$ 3.00 / \mathrm{m}$ (for dipoles)
$=\$ 3.66 \times 10^{6}$

1d. From Part II:
$\left(2 \times 10^{5} \mathrm{~m}\right) \$ 3.00 / \mathrm{m}$ (for quadrupoles)
$=\$ 0.6 \times 10^{6}$

2a. Cryogenic cost owing to ordinary heat leaks will be that for an ordinary Dewar and is estimated to be about $\$ 2000 / \mathrm{m}$ of circumference.

For the pulsed accelerator $=2 \pi p$ (circumference factor) $\$ 2000$

$$
=2 \pi p(1.4) \$ 2000=\$ 2.9 \times 10^{6}
$$

For the FFAG accelerator $=2 \pi \rho(2) \quad \$ 2000=\$ 5 \times 10^{6}$
2b. At present it is observed ${ }^{8}$ that the energy loss owing to pulsing superconductors is given by:
Energy loss (J/cycle) $\equiv 0.5 \%$ of the maximum stored energy in the system.
Also, it might be possible in the future to reduce this loss by a factor of 5 so that to be optimistic the future $10 s s$ would be only $0.1 \%$.
Also note that at the present time refrigerating equipment costs about $\$ 400 / \mathrm{W}$ at 500 W and at $4^{\circ} \mathrm{K}$. In the future, according to the consensus of this Summer Study, this might be reduced to $\$ 100 / \mathrm{W}$.

For these calculations it is assumed that the accelerator magnets will be pulsed once in 4 sec. Consequently the cost
i. Present $=\$ 100 \times 10^{6}$
ii. Future $=\$ 5 \times 10^{6}$.

This gives such a wide spread in cost that no reliable conclusion can be made. However, for the next few years an average value of the present and future price may not be far wrong.
3. This is based on the present cost for the new Brookhaven generator which is $\$ 2 \times 10^{6}$ to handle $12 \times 10^{6} \mathrm{~J}$ of stored energy. Then for the $2 \times 10^{8} \mathrm{~J}$ to be stored here the scaled cost is $\$ 30 \times 10^{6}$. But P.F. Smith ${ }^{9}$ at this Summer Study has expressed the belief that a superconducting power supply could be made in the future for about one-half of the present cost. Much engineering and development needs to be done before this is accomplished.

[^0]5. The vacuum envelope can be just a stainless-steel tube or the equivalent (it is cooled to $4^{\circ} \mathrm{K}$ ) and so with pumps, valves, controls, etc. may cost $\$ 400$ or $\$ 500 / \mathrm{m}$.
6. The ring housing (tunnel) for the magnet is assumed to cost about $\$ 3000 / \mathrm{m}$.
7. Miscellaneous engineering costs: coil windings, structural support, outer shell, etc. Guess $\$ 500 / \mathrm{m}$.

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G. Parzen of Brookhaven.


Fig. 1. Spiral-separated sectors for an FFAG model accelerator at Brookhaven, drawn by D. Jacobus of Brookhaven. The dimensions are somewhat different from those used in the text but all essential features are as Parzen originally outlined them.


Fig. 2. Details of one spiral sector as drawn by D. Jacobus of Brookhaven. The number of turns of 0.5 in. wide $\mathrm{Nb}_{3} \mathrm{Sn}$ per subsection is different from that used in the text, as are the radii of the semicircular ends (see Table I). However, the general features of the construction are maintained and are pertinent.


Fig. 3. Schematic coil construction to simulate actual coils such as those in Figs. 1 and 2. This geometry is assumed in the magnetic field calculations.


Fig. 4. Calculated $B(k G)$ vs $R(c m)$ at $\theta=30.5^{\circ}$. The current in the 0.5 in . wide $\mathrm{Nb}_{3} \mathrm{Sn}$ ribbon is assumed to be 2000 A . Figure 4 a shows the unbiased field; Fig. 4b shows the biasing field; Fig. $4 c$ shows the net biased field, i.e., the sum of the unbiased and biasing field.


SCHEMATIC FLUX BIAS: ELLIPSE $\frac{a}{b}=\frac{3}{1}$
Fig. 5. Schematic flux biasing coils. The flux biasing coils are placed on an elliptical cross section which surrounds the main field coils. The net biased field is perpendicular to the median plane of the accelerator and vacuum chamber (see text).


Fig. 6. Magnetic field (kG) plotted vs the radius (cm) of the accelerator for four values of $\theta$. The peaks of magnetic field owing to the spiral ridges are clearly resolved and appear at successively larger radii as they should in a good FFAG accelerator.


Fig. 7. Log of $B$ vs R. Data used to calculate $k$, the field index, from $R=239 \mathrm{~cm}$ to $R=261 \mathrm{~cm}$.


Fig. 8. Schematic details of a spiral sector for an FFAG superconducting 300 GeV accelerator (not to scale).

# PRELTMINARY STEPS FOR APPLYING <br> SUPERCONDUCTORS TO FFAG ACCELERATORS* 

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## INTRODUCTION

In our effort to see the possibility of applying superconductors to accelerators, using magnetic guide fields, we came up with several questions. For some of them we have an answer, for others we see probable solutions; but there are still several for which we do not see a workable solution. Although this is our present situation, our answers and doubts are presented in this paper.

Some general considerations will be made on the problem of building a superconducting ring for an accelerator. These considerations will apply only to the guide field; problems related to injection and extraction or rf acceleration, although very important, are not discussed in this paper. Rather than presenting a detailed study of a cryogenic system with optimum performance, emphasis is placed on those problems where clear solutions are needed if one thinks seriously of building an accelerator.

Without talking specifically of a particular type of accelerator (pulsed or dc), we will discuss in general the problems of accuracy in field measurements, in positioning and aligmment of a magnet set subject to thermal contraction and strong magnetic forces.

In what follows, three possible ways of building an accelerator magnet ring are presented.
(A). The whole accelerator is in a vacuum chamber which serves as a cryogenic enclosure. This scheme is shown in Fig. 1. This design will apply to small machines, perhaps not more than 4 m diam; beyond this size this design is not economical.
If the magnets, forming the ring, have no obstructions in the median plane (open midplane), field measuring and positioning can be made inside of the enclosure at $4.2^{\circ} \mathrm{K}$. It is required to have a support for the field measuring instruments and a boom that can move radially and azimuthally. The field mapping will be in polar coordinates.
One can conceive ways of "zeroing" the measuring scales at $4.2^{\circ} \mathrm{K}$ before starting to take magnetic measurements. A proper design may allow the field measuring devices to remain inside of the enclosure for future checking of the fields. Notice that this can be accomplished easily as there is no vacuum chamber in the field region.
For closed mid-plane magnets, the previous procedure will not apply. Magnetic measurements need to be carried out on a separate cryogenic measuring chamber (see Fig. 12); once a magnet is "measured" it takes its place in the ring. One very important point in this operation is that positioning a magnet requires reference marks that are related to magnetic measurements; the operation

[^1]of the magnets is at $4.2^{\circ} \mathrm{K}$; thus, to minimize errors, a correspondence between field values and coordinates should be established at such a temperature. We will come back to discuss this point further.
(B). The second alternative corresponds to the case of an accelerator having all the magnets in the ring sharing a common vacuum-cryogenic chamber as well as the same mechanical support. This case is illustrated in Fig. 2. This design will be suitable for large machines.

One can visualize at least two possible ways for field measuring and positioning of the magnets.
(i). Using a cryogenic measuring chamber as mentioned above, regardless of whether the mid-plane is open or closed.- After measuring each magnet it is transferred into the ring. The final alignment of all magnets should be made at $4.2^{\circ} \mathrm{K}$; thus, it is essential to have access to reference marks or targets through the cryogenic enclosure. Azimuthal positioning can be done by normal surveying techniques and radial positioning by laser interferometry.
(ii). Using a field measuring machine that can be part of the ring.- A magnet (and modular vacuum-cryogenic chamber) can be roughly positioned in the ring, the enclosure evacuated, cooled down, field measurements taken, and the reference marks established for future accurate positioning. In this case, as before, the reference marks should be accessible, through a window, from outside of the enclosure. Successive magnets will be accommodated in the ring until closed. Although this procedure may have some advantages, a detailed economic study will be needed to decide its applicability.

Versions $A$ and $B$ are also attractive from the point of view that, having the vacuum chamber at $4.2^{\circ} \mathrm{K}$, ideal conditions against gas scattering will be obtained.
(C). The field region of the magnet is at room temperature, each magnet being a separate unit (see Fig. 3). This design may be suitable for a very large machine, where each magnet is far away from its neighbor. In this last option, the minimum separation between two adjacent magnets is determined by the magnitude of the magnetic force that can be tolerated without producing misalignments.
The advantage of this design is that the magnetic fields can be measured, using well-known techniques, at room temperature; however, positioning reference marks should be located in the magnet, not in the shell of the cryogenic enclosure that, by definition, cannot be rigidly attached to the magnet.
This design may require a separate vacuum chamber, whereas in the two previous ones the cryogenic enclosure provides the vacuum needed for thermal insulation as well as for the beam. As mentioned before, the absence of a vacuum chamber may allow us to measure magnetic fields, at least in some spots, when the whole magnet ring is already assembled. This was a practice followed in the past for FFAG accelerators using iron magnets:

The previous examples illustrate the types of problems that will be encountered in this field. Neither of these designs presents a complete picture of the situation, as many details have been omitted for simplicity. Satisfactory solutions require the development of a new technology to meet tight tolerances in an environment at $4.2^{\circ} \mathrm{K}$. A possible way to find some solutions will be outlined in this paper.


[^0]:    7. A private communication estimates the future cost of 1.0 in. wide ribbon at 4000 A as $\$ 6.00 / \mathrm{m}$ in quantities of two million meters or more for delivery over 1 to 2 years. This is equivalent to 0.5 in. ribbon at $\$ 3.00 / \mathrm{m}$.
    8. W.B. Sampson, private communication.
    9. P.F. Smith, these Proceedings, p. 1002.
[^1]:    ${ }^{\star}$ Work performed under the auspices of the U.S. Atomic Energy Commission.

