Harmonic Coils

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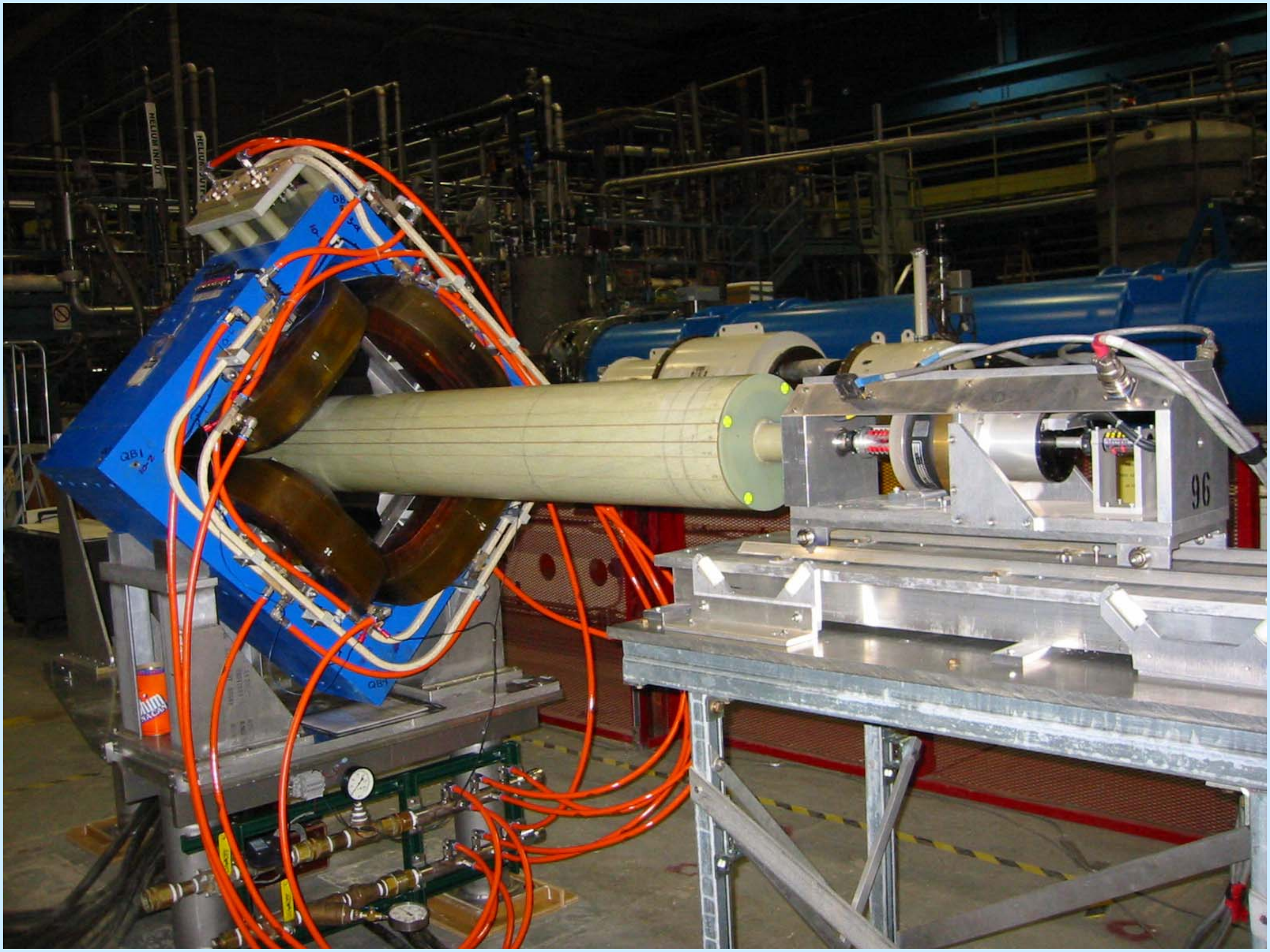
Introduction

- For most accelerator magnets, a harmonic description of the field is often used, both for characterizing the field quality, as well as for particle tracking studies.
- The "Harmonic Coil" technique, employing rotating coils, is the most convenient, accurate, and widely used technique for the measurement of harmonic coefficients in accelerator magnets.

Basic Principle

- The harmonic coefficients are related to the azimuthal variation of the field components.
- A rotating coil (loop of wire) measures the azimuthal variation of the intercepted flux. The field harmonics are then deduced using a knowledge of the geometry of the coil.
- A coil often uses several loops of different geometries to improve the accuracy of measurements by a process of ***"bucking"***.

A Typical Rotating Coil Setup

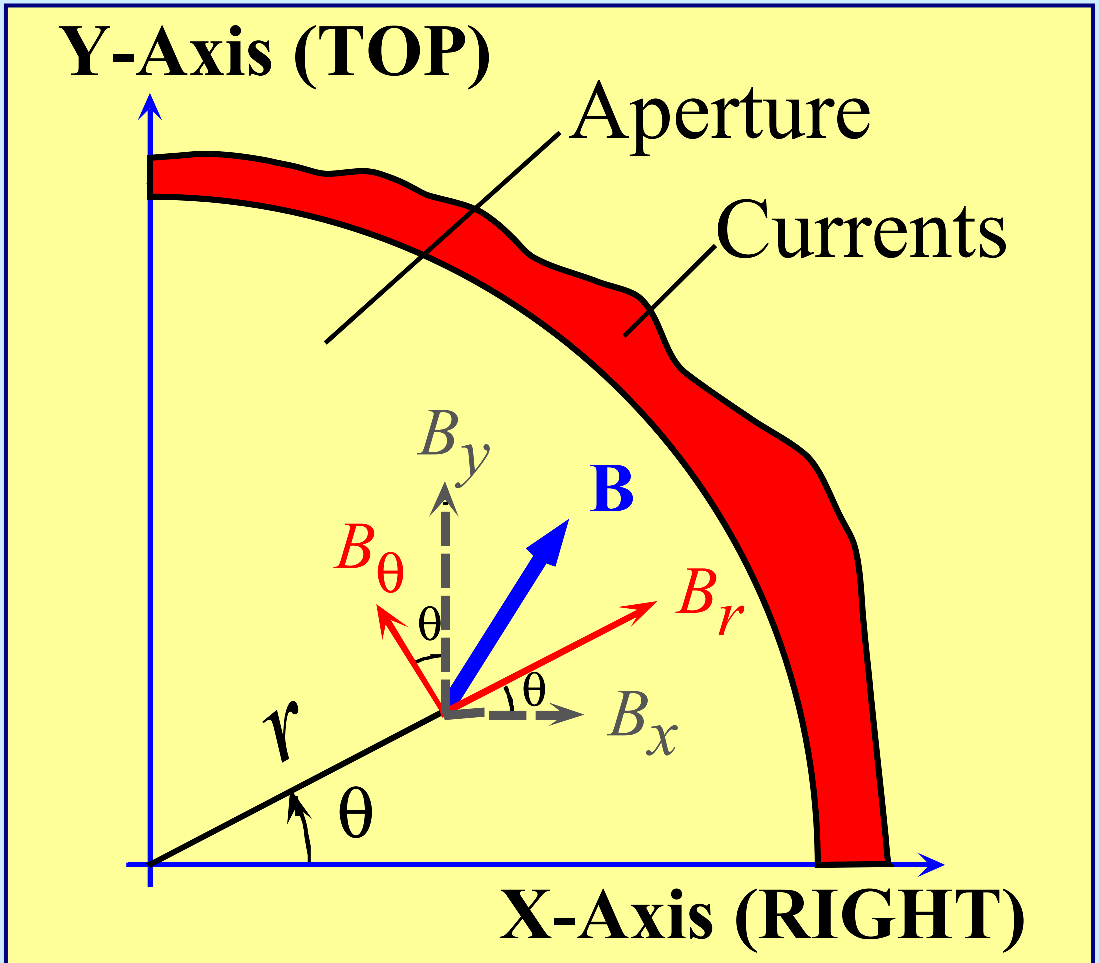


Common Coil Geometries

- All geometries employ a loop of wire, with one pair of sides parallel to the magnet axis.
- The plane of the loop can be oriented in an arbitrary direction, but two specific geometries, known as ***"radial"*** or ***"tangential"*** coils, are most common due to ease of fabrication, characterization and data analysis.
- Special geometries to measure specific harmonics are also used.

Useful Background Information

Commonly Used Coordinate System



View from the Lead End of the Magnet

Users of magnetic measurements data may use a system oriented differently, often requiring suitable transformations of the measured harmonics.

Useful Background Information

Expressions for the Field Components

$$B_r(r,\theta) = \sum_{n=n_0}^{\infty}\left[B_n \sin\{(n-n_0+1)\theta\} + A_n \cos\{(n-n_0+1)\theta\}\right]\left(\frac{r}{R_{ref}}\right)^{n-n_0}$$

$$B_\theta(r,\theta) = \sum_{n=n_0}^{\infty}\left[B_n \cos\{(n-n_0+1)\theta\} - A_n \sin\{(n-n_0+1)\theta\}\right]\left(\frac{r}{R_{ref}}\right)^{n-n_0}$$

$$B_y + iB_x = \sum_{n=n_0}^{\infty}\left[B_n + iA_n\right]\left(\frac{x+iy}{R_{ref}}\right)^{n-n_0}$$

$n_0 = 0$: US

$n_0 = 1$: European

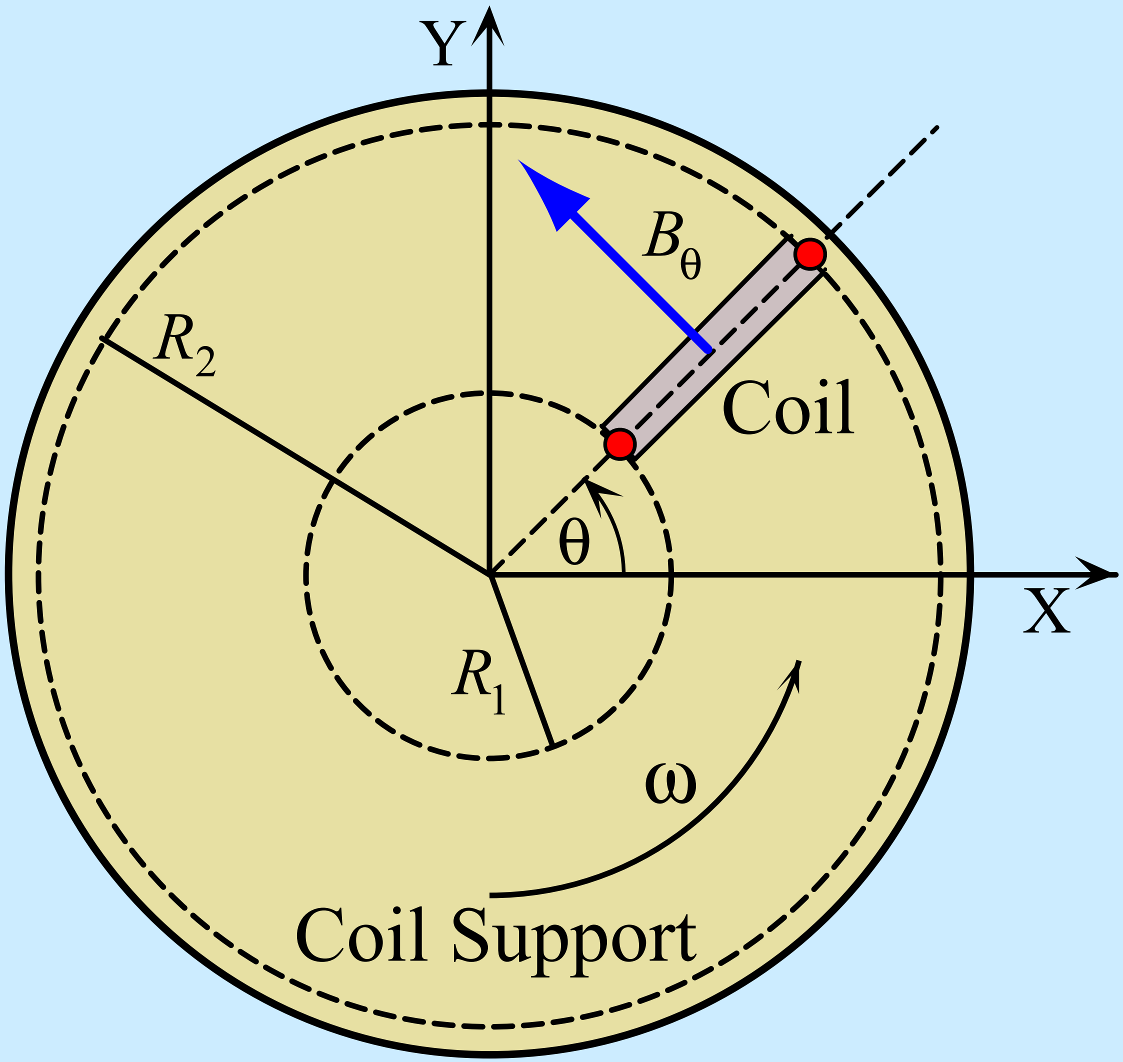
R_{ref} = Reference radius

B_{n+n_0-1} = $2n$ - pole NORMAL Term

A_{n+n_0-1} = $2n$ - pole SKEW Term

(n_0=1 followed in this talk)

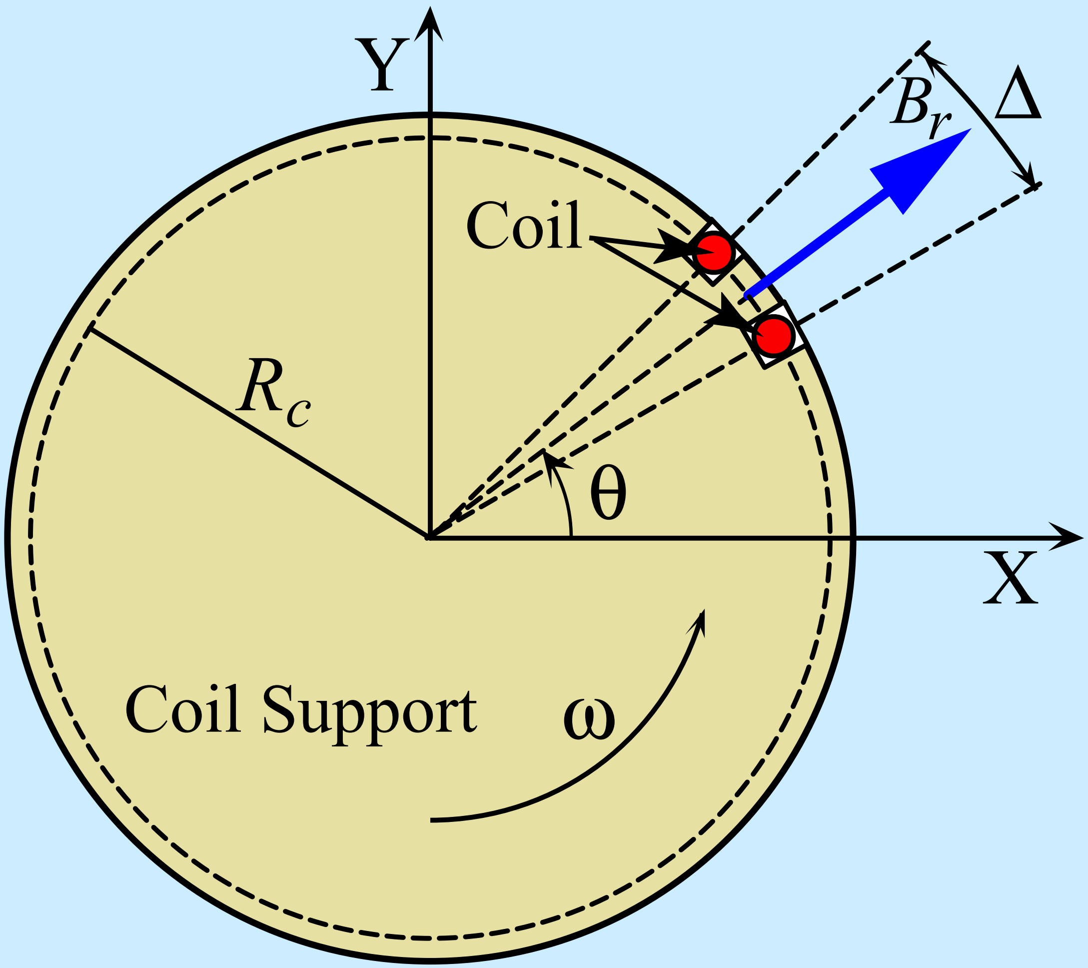
Cross Section of a Radial Coil



Loop of wire is placed in a radial plane to produce a RADIAL COIL.

A radial coil is sensitive to the AZIMUTHAL component of the field

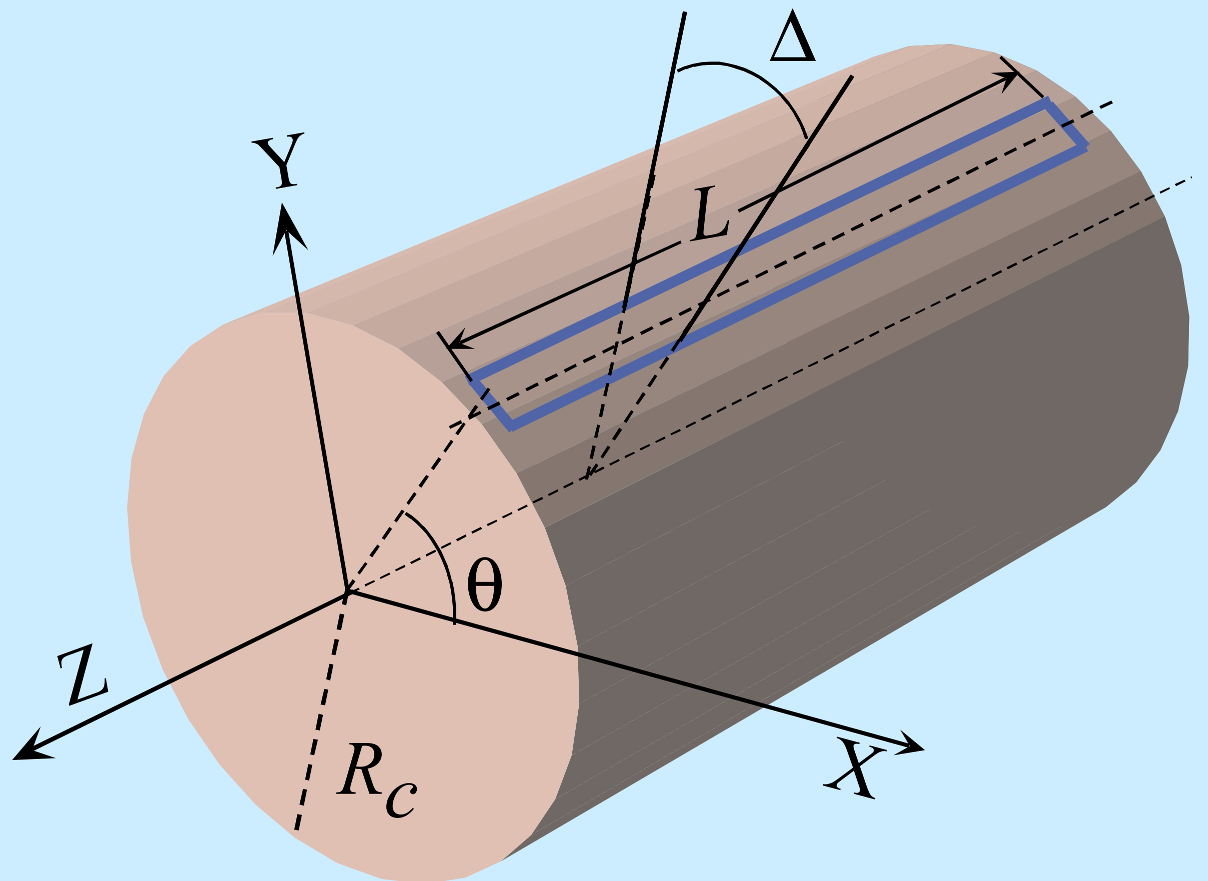
Cross Section of a Tangential Coil



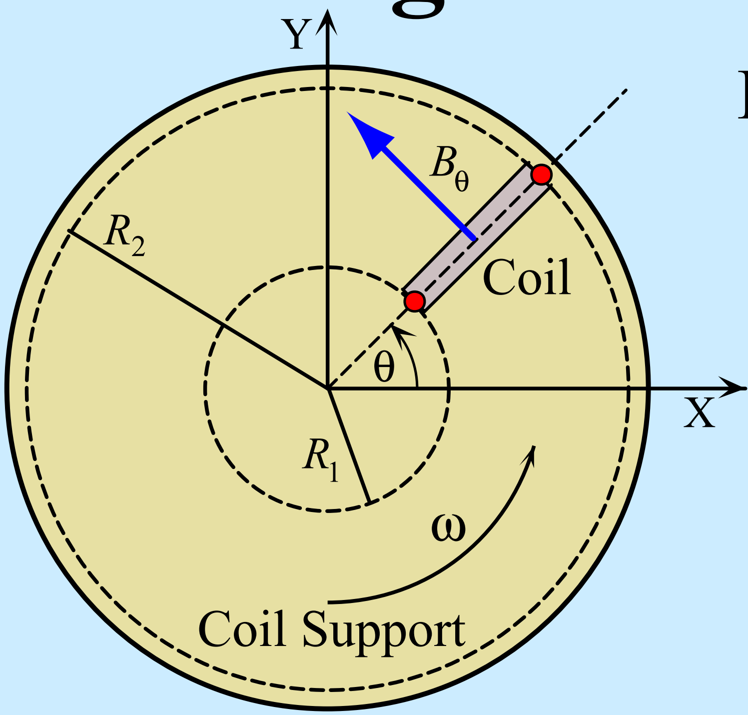
Loop of wire is placed NORMAL to the radius vector to produce a TANGENTIAL COIL.

A tangential coil is sensitive to the RADIAL component of the field

3-D View of a Tangential Coil



Signal from a Radial Coil



Flux through the coil at time t is:

$$\Phi(t) = NL\int_{R_1}^{R_2} B_\theta(r,\theta)\,dr$$

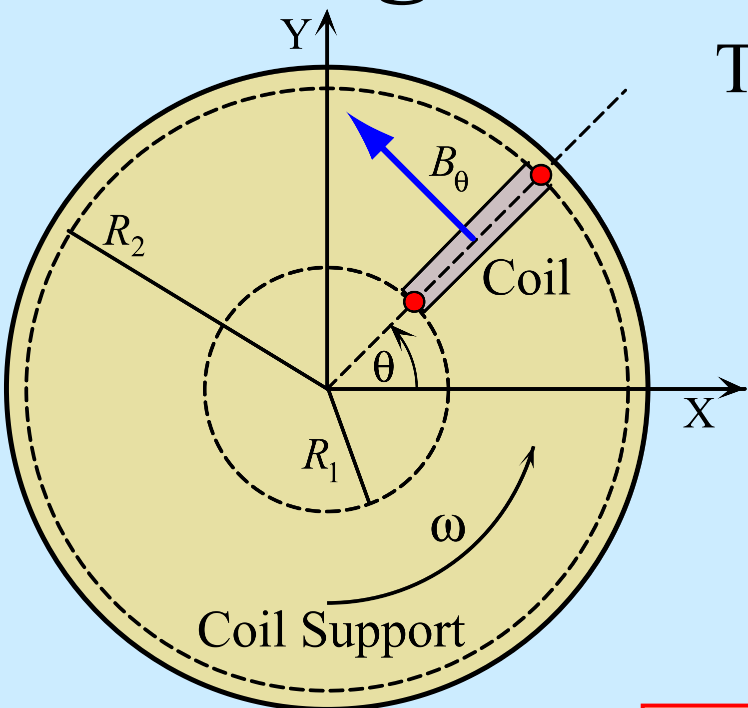
$$= \sum_{n=1}^{\infty} \frac{NLR_{ref}}{n}\left[\left(\frac{R_2}{R_{ref}}\right)^n - \left(\frac{R_1}{R_{ref}}\right)^n\right]\times$$

$$\left[B_n\cos(n\omega t + n\delta) - A_n\sin(n\omega t + n\delta)\right]$$

N = No. of turns
L = Length
δ = angle at (t = 0)
ω = angular velocity
$\theta = \omega t + \delta$ (angle at t)

The periodic variation of flux is described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, and geometric parameters of the coil.

Signal from a Radial Coil



The voltage signal at time t is:

$$V(t) = -\left(\frac{d\Phi}{dt}\right)$$

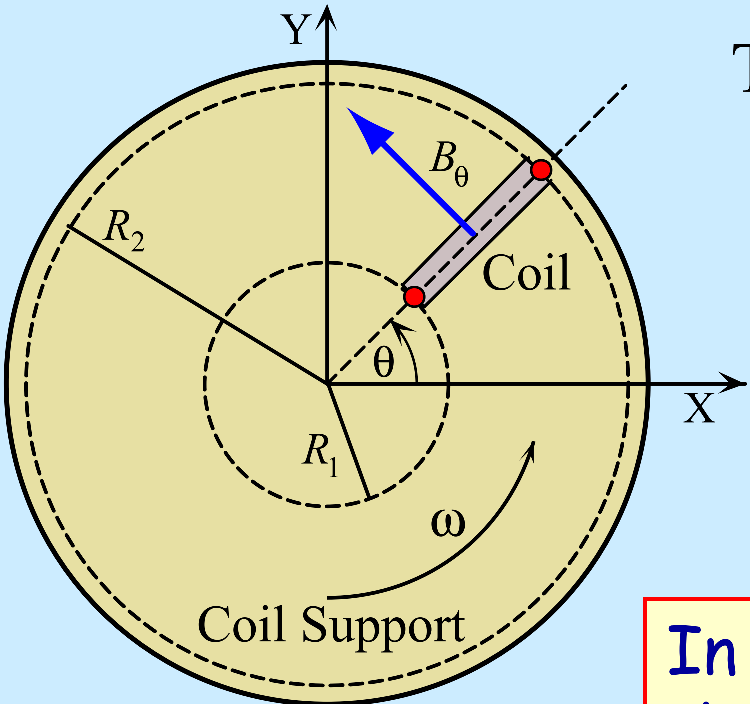
$$= \sum_{n=1}^{\infty} NLR_{ref}\,\omega \left[\left(\frac{R_2}{R_{ref}}\right)^n - \left(\frac{R_1}{R_{ref}}\right)^n\right] \times$$

$$\left[B_n \sin(n\omega t + n\delta) + A_n \cos(n\omega t + n\delta)\right]$$

N = No. of turns
L = Length
δ = angle at ($t = 0$)
ω = angular velocity
$\theta = \omega t + \delta$

The periodic variation of coil voltage is also described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, the geometric parameters of the coil, and angular velocity.

Radial Coil Analysis: Voltmeters



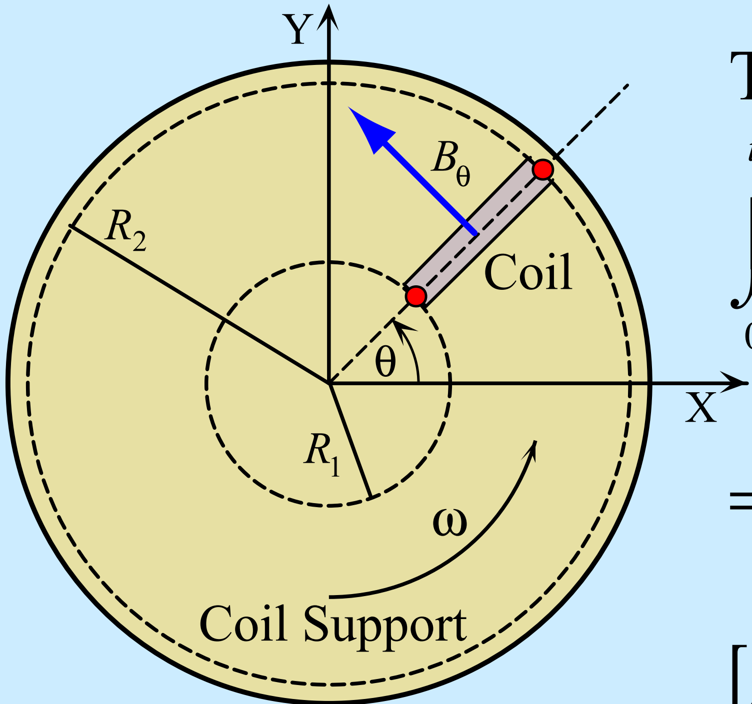
The voltage signal at time t is:

$$V(t) = \sum_{n=1}^{\infty} NLR_{ref}\,\omega \left[\left(\frac{R_2}{R_{ref}} \right)^n - \left(\frac{R_1}{R_{ref}} \right)^n \right] \times$$
$$\left[B_n \sin(n\omega t + n\delta) + A_n \cos(n\omega t + n\delta) \right]$$

N = No. of turns
L = Length
δ = angle at ($t = 0$)
ω = angular velocity
$\theta = \omega t + \delta$

In the voltmeter mode, it is essential that the coil rotational speed be well controlled (typically ~0.1%). In addition, the raw data may have to be rescaled to the as-measured instantaneous speed for improved accuracy.

Radial Coil Analysis: Integrators



The integrated voltage at time t is:

$$\int_0^t V(t)dt = \Phi(0) - \Phi(\theta)$$

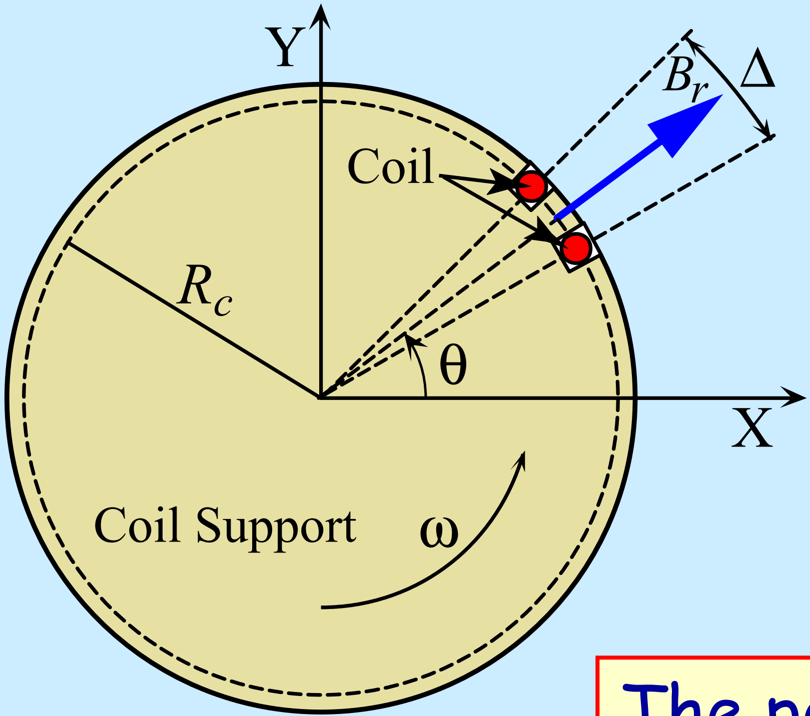
$$= \Phi(0) - \sum_{n=1}^{\infty} \frac{NLR_{ref}}{n} \left[\left(\frac{R_2}{R_{ref}} \right)^n - \left(\frac{R_1}{R_{ref}} \right)^n \right] \times$$

$$\left[B_n \cos(n\theta + n\delta) - A_n \sin(n\theta + n\delta) \right]$$

N = No. of turns
L = Length
δ = angle at (t = 0)
ω = angular velocity
$\theta = \omega t + \delta$

The integrator mode has the advantage that the signal is independent of the rotational speed. The integrator drift, however, can be a problem, and needs correction.

Signal from a Tangential Coil



Flux through the coil at time t is:

$$\Phi(t) = NL \int_{\theta-\Delta/2}^{\theta+\Delta/2} B_r(R_c,\theta) R_c \, d\theta$$

$$= \sum_{n=1}^{\infty} \frac{2NLR_{ref}}{n} \left(\frac{R_c}{R_{ref}}\right)^n \sin\left(\frac{n\Delta}{2}\right) \times$$

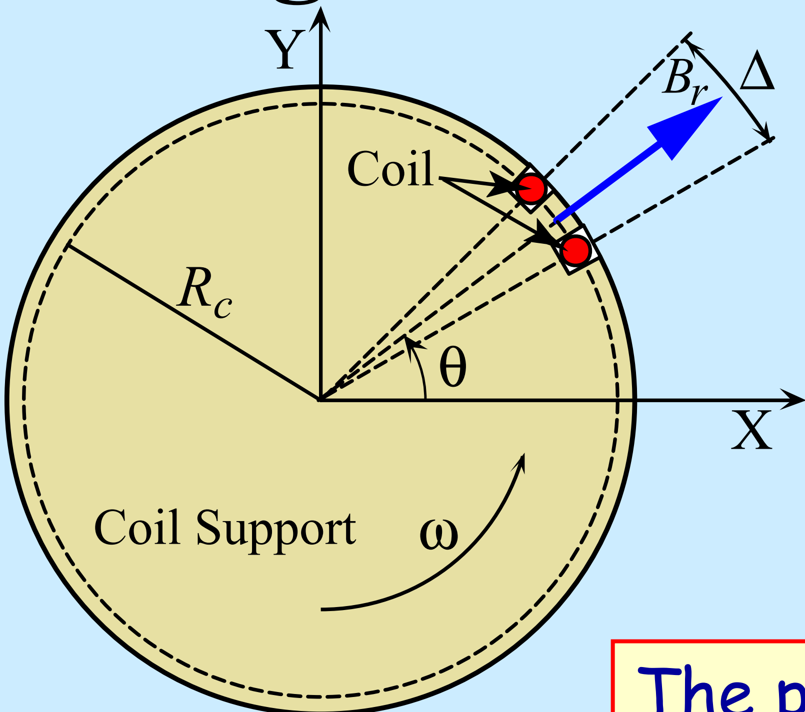
$$\left[B_n \sin(n\omega t + n\delta) + A_n \cos(n\omega t + n\delta)\right]$$

N = No. of turns
L = Length
Δ = Opening angle
δ = angle at (t = 0)
ω = angular velocity
$\theta = \omega t + \delta$ (angle at t)

The periodic variation of flux is described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, and geometric parameters of the coil.

Note: Phase different from Radial Coil.

Signal from a Tangential Coil



The voltage signal at time t is:

$$V(t) = -\left(\frac{d\Phi}{dt}\right)$$

$$= \sum_{n=1}^{\infty} 2NLR_{ref}\,\omega \left(\frac{R_c}{R_{ref}}\right)^n \sin\left(\frac{n\Delta}{2}\right) \times$$

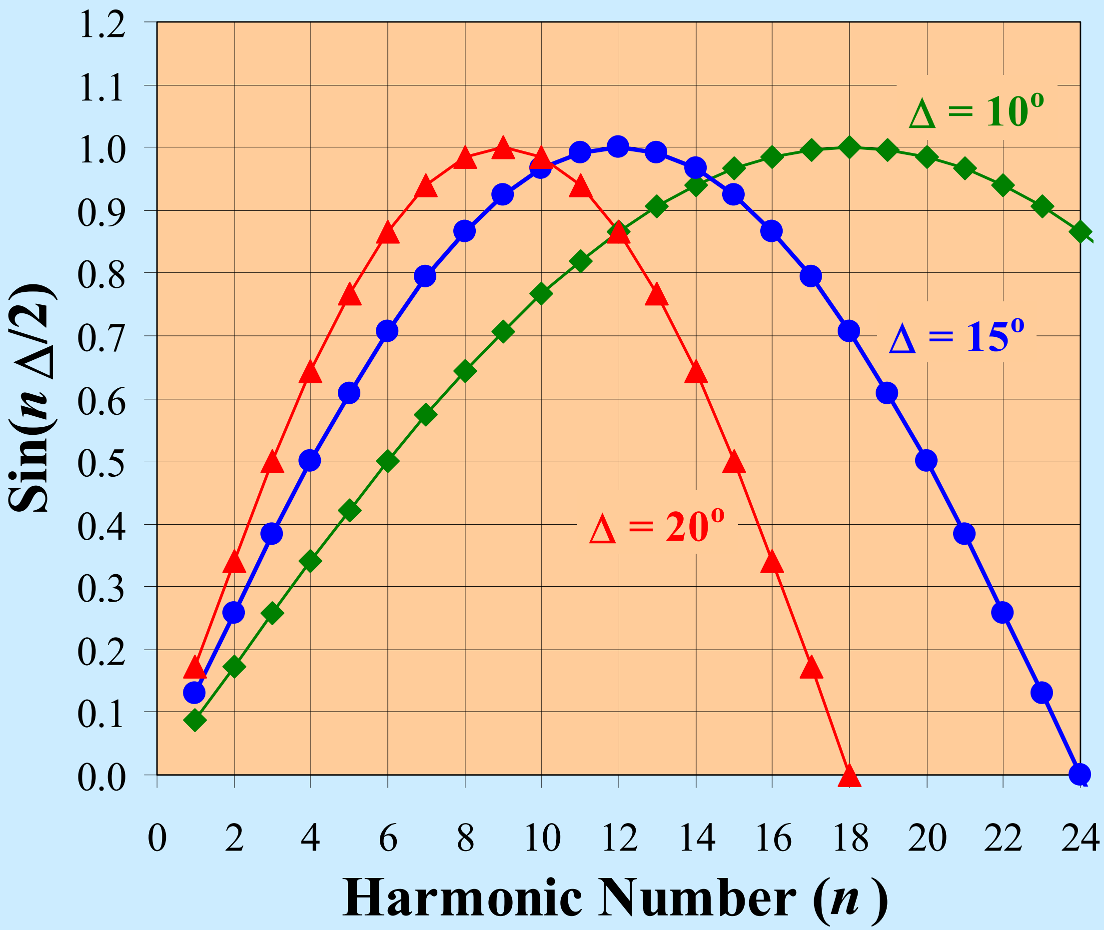
$$\left[A_n \sin(n\omega t + n\delta) - B_n \cos(n\omega t + n\delta)\right]$$

N = No. of turns
L = Length
Δ = Opening angle
δ = angle at (t = 0)
ω = angular velocity
$\theta = \omega t + \delta$ (angle at t)

The periodic variation of coil voltage is also described by a Fourier series, whose coefficients are related to the Normal and Skew harmonics, the geometric parameters of the coil, and angular velocity.

Note: Phase different from radial coil.

Optimal Choice of Opening Angle

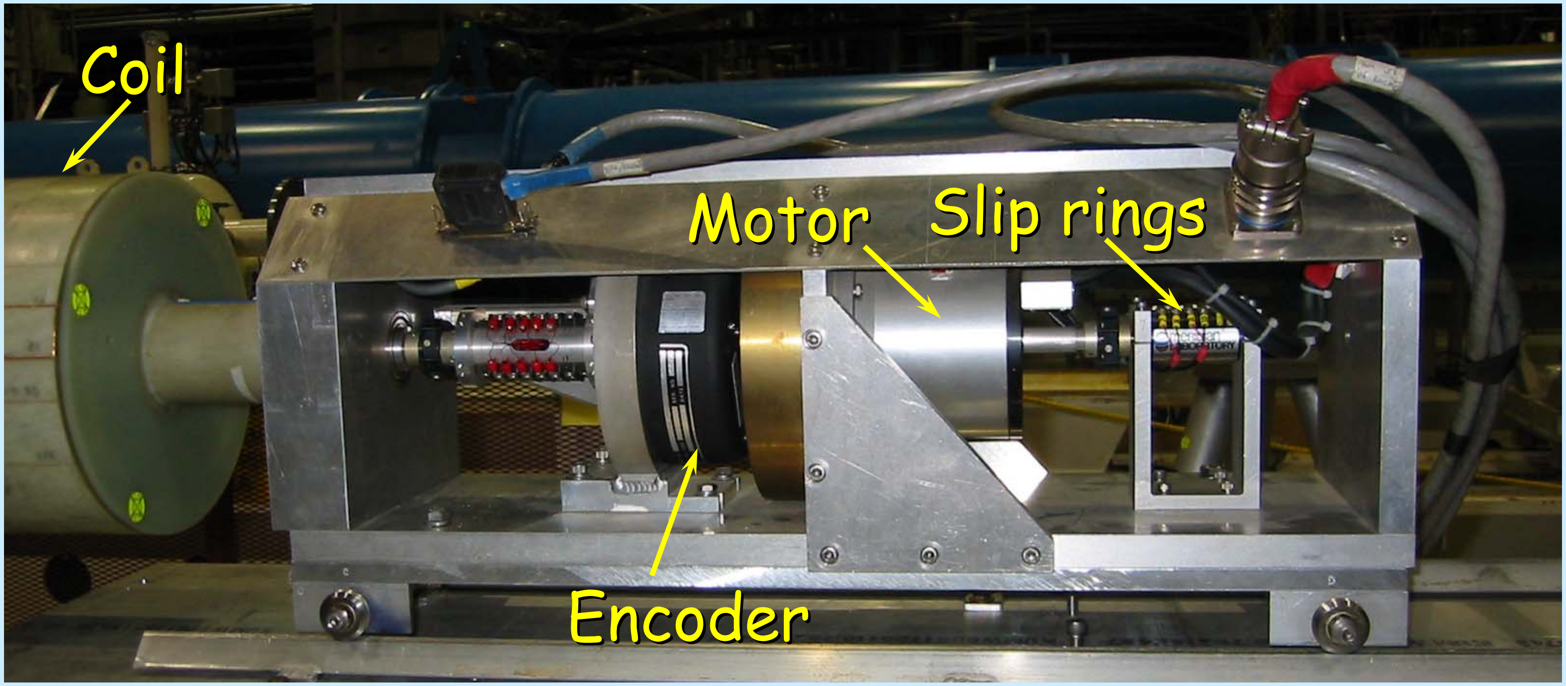


Typically, one is interested in about 15 harmonics.

Too large an angle loses sensitivity for higher harmonics.

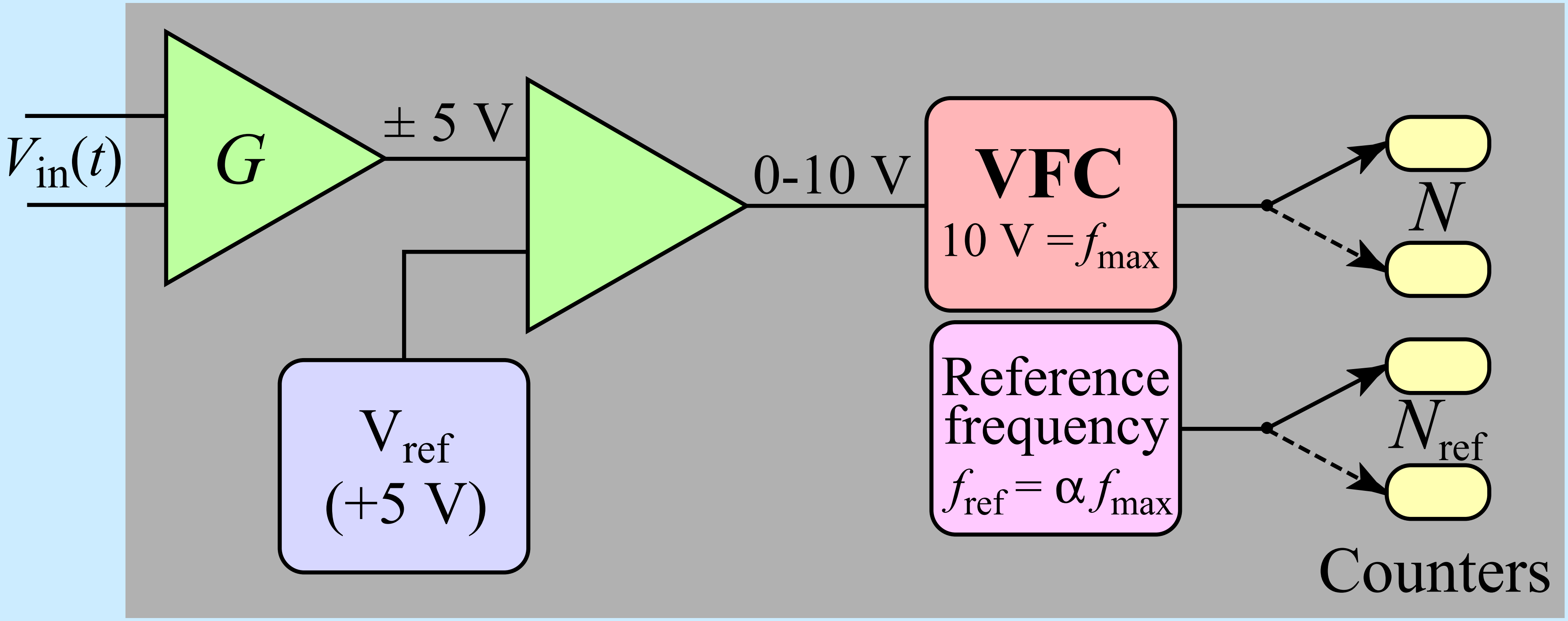
Too small an angle sacrifices sensitivity to lower harmonics of most interest.

Acquisition of Rotating Coil Data



The output of the coil can be connected to either **precision voltmeters**, or to **digital integrators**. The rotating shaft of the coil is coupled to an **angular encoder**, which generates triggers at uniform angle intervals for storing the data. The time between triggers can be measured and used for stabilization of the speed, if necessary.

Digital Integrator (CERN/Metrolab)



Dual Counters: frequency output can be switched to the second pair while the first pair is being read.

$$\int_0^t V_{in}(t)\,dt = \left(\frac{10}{G\,f_{max}}\right)\left[N - \frac{N_{ref}}{2\alpha}\right]$$

$$\text{Resolution} = \left(\frac{10}{G\,f_{max}}\right)\text{Volt second}$$

Digital Voltmeters

HP 3457

Full Scale: 30 mV to 300V

Integration time:
0.0005 to 100 power cycles

Resolution: 3.5 to 7.5 digits

Max. reading rate: 1.35 kHz

HP 3458A

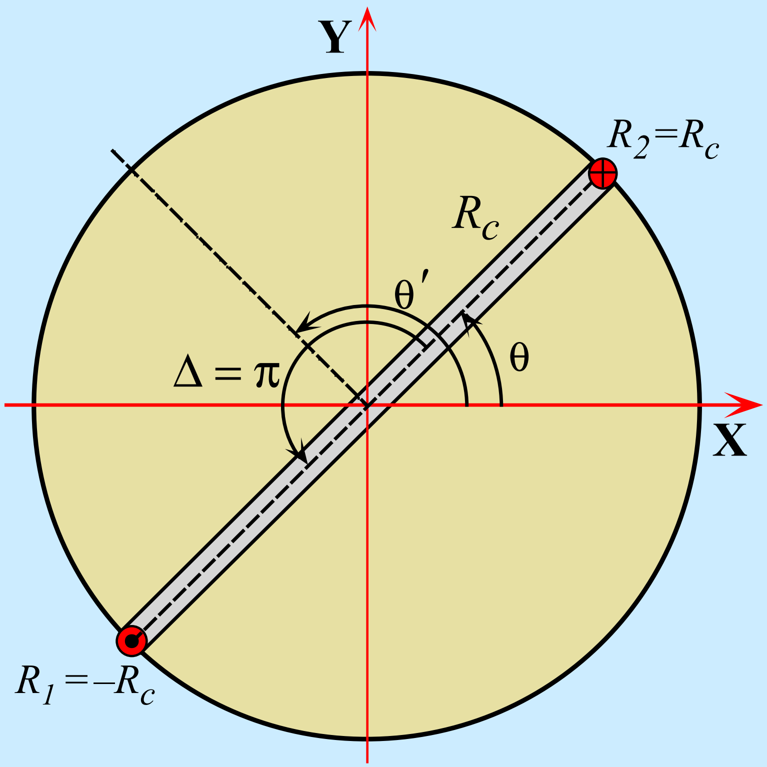
Full Scale: 100 mV to 1000V

Integration time:
0.0001 to 1000 power cycles

Resolution: 4.5 to 8.5 digits

Max. reading rate: 100 kHz

Special Geometries: A "Dipole Coil"



A "Dipole Coil" can be viewed as a tangential coil of opening angle 180°. Such a coil is sensitive only to the **dipole, sextupole, ...** harmonics. It is commonly used to "buck" the main field term in dipole magnets.

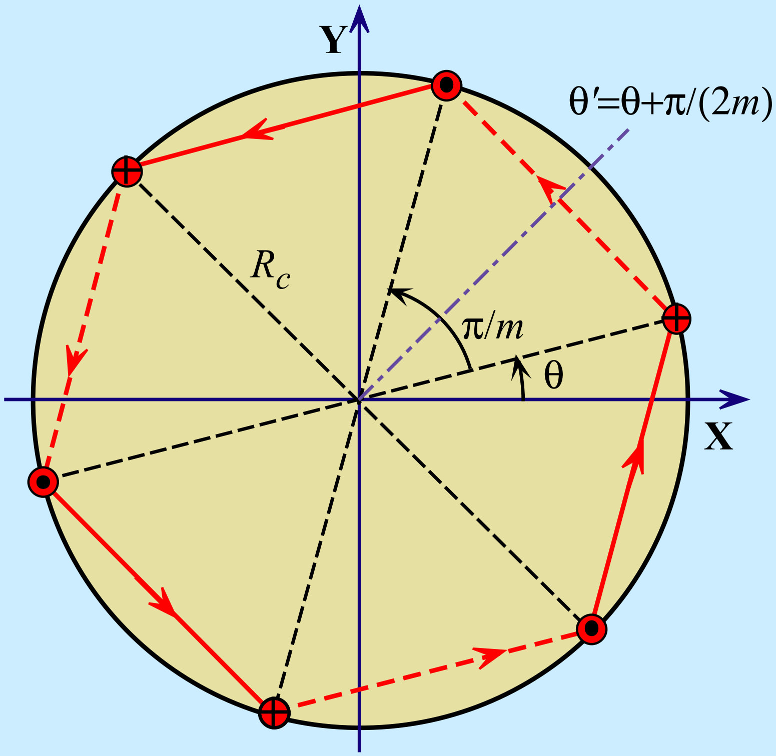
N = No. of turns
L = Length
Opening angle = 180°
θ = angular position

Flux through the coil at position θ:

$$\Phi_{Dipole}(\theta) = \sum_{\substack{n=1 \\ n=odd}}^{\infty} \frac{2NLR_{ref}}{n} \left(\frac{R_c}{R_{ref}} \right)^n \times \left[B_n \cos(n\theta) - A_n \sin(n\theta) \right]$$

Note: Angle θ is to the plane of the winding, not the centerline angle.

Special Geometries: A "2*m*-pole Coil"



A "2*m*-pole Coil" can be viewed as an array of *m* tangential coils in series, each of opening angle π/m, and located at every $2\pi/m$ radians. Such a coil is sensitive only to those harmonics which are **odd multiples of *m***.

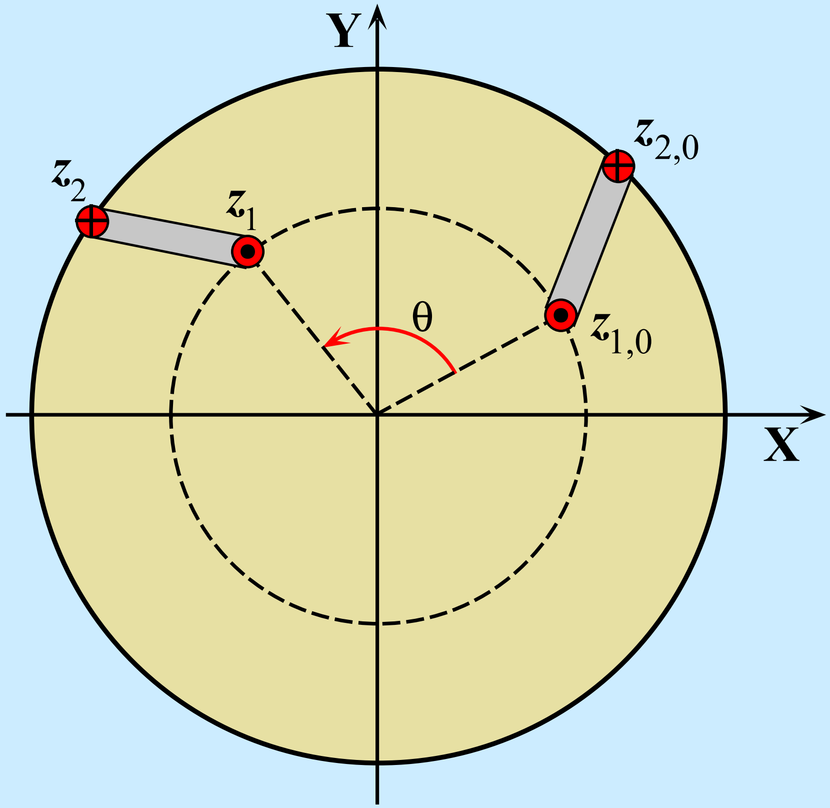
N = No. of turns
L = Length
Opening angle = π/m
θ = angular position
m equispaced coils in series.

Flux through the coil at position θ:

$$\Phi(\theta) = \sum_{\substack{n=m \\ n=(2k+1)m}}^{\infty} \frac{2mNLR_{ref}}{n}\left(\frac{R_c}{R_{ref}}\right)^n \times \left[B_n\cos(n\theta) - A_n\sin(n\theta)\right]$$

Note the definition of angle θ.

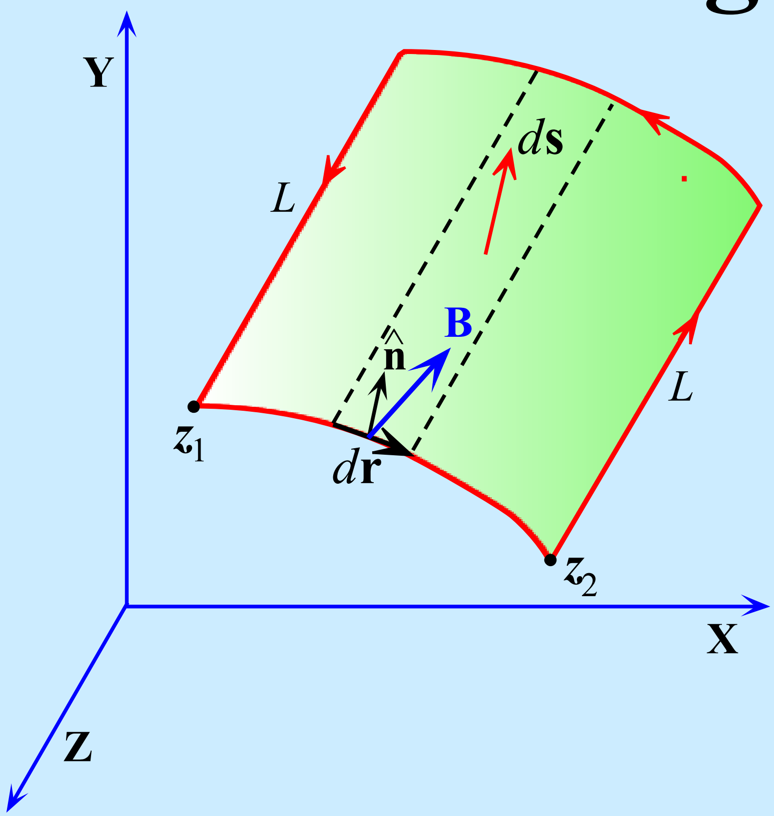
A Generalized Rotating Coil



In general, the loop may be placed in an arbitrary orientation. Such a loop is sensitive to both the radial and tangential components. The characterization and data analysis for such a loop is relatively more complex.

In practice, ALL coils have some sensitivity to BOTH field components due to construction errors. This may become important, particularly when using the same coil for precise measurements of field direction of several multipoles.

Flux Through an Arbitrary Coil



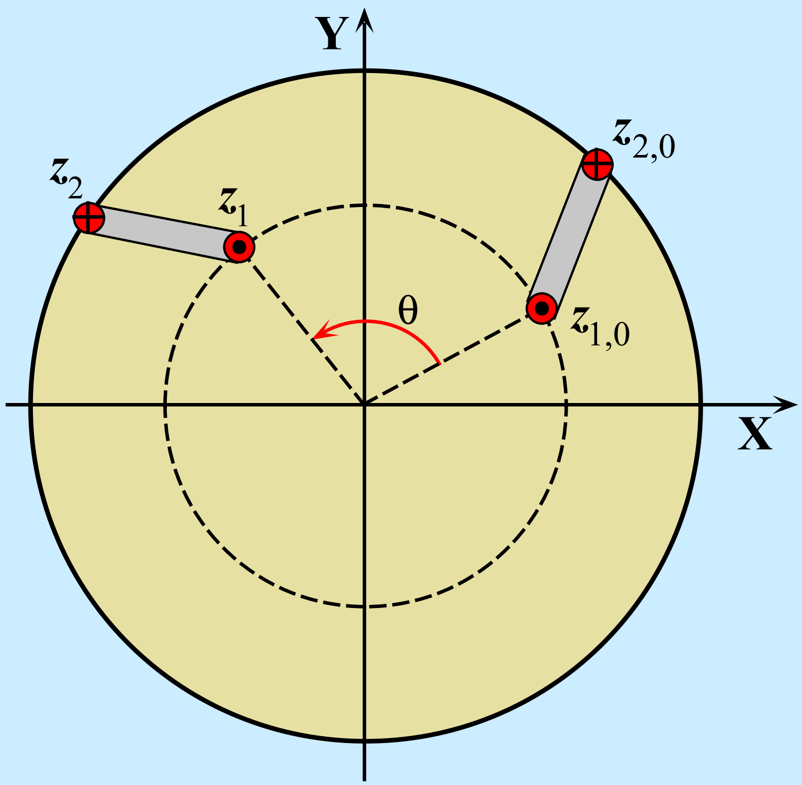
$$d\Phi = N \cdot \mathbf{B}.d\mathbf{s} = N \cdot (B_y dx - B_x dy)L$$
$$= NL \cdot \mathrm{Re}\left[(B_y + iB_x)(dx + idy)\right]$$
$$= NL \cdot \mathrm{Re}\left[\boldsymbol{B}(\boldsymbol{z})d\boldsymbol{z}\right]$$

$\boldsymbol{z} = x + iy$; it is <u>not</u> the axial-coordinate.

$$\Phi = NL\,\mathrm{Re}\left[\int_{z_1}^{z_2} \boldsymbol{B}(\boldsymbol{z})d\boldsymbol{z}\right]$$
$$= \mathrm{Re}\left[\sum_{n=1}^{\infty}\left(\frac{NLR_{ref}}{n}\right)\left\{\left(\frac{\boldsymbol{z}_2}{R_{ref}}\right)^n - \left(\frac{\boldsymbol{z}_1}{R_{ref}}\right)^n\right\}\left(B_n + iA_n\right)\right]$$

For 2-D fields, the flux depends only on the points z_1 and z_2 of the loop. It does not depend on how the loop is closed from z_1 to z_2. The flux for radial and tangential coils can be derived as special cases of this general result.

Sensitivity Factor of a Coil



After a rotation of the coil by θ:

$$z_1 = z_{1,0}\exp(i\theta); \quad z_2 = z_{2,0}\exp(i\theta)$$

$$\Phi(\theta) = \mathrm{Re}\left[\sum_{n=1}^{\infty} \boldsymbol{K}_n \exp(in\theta)\,(B_n + iA_n)\right]$$

where $\boldsymbol{K}_n$ is the sensitivity factor:

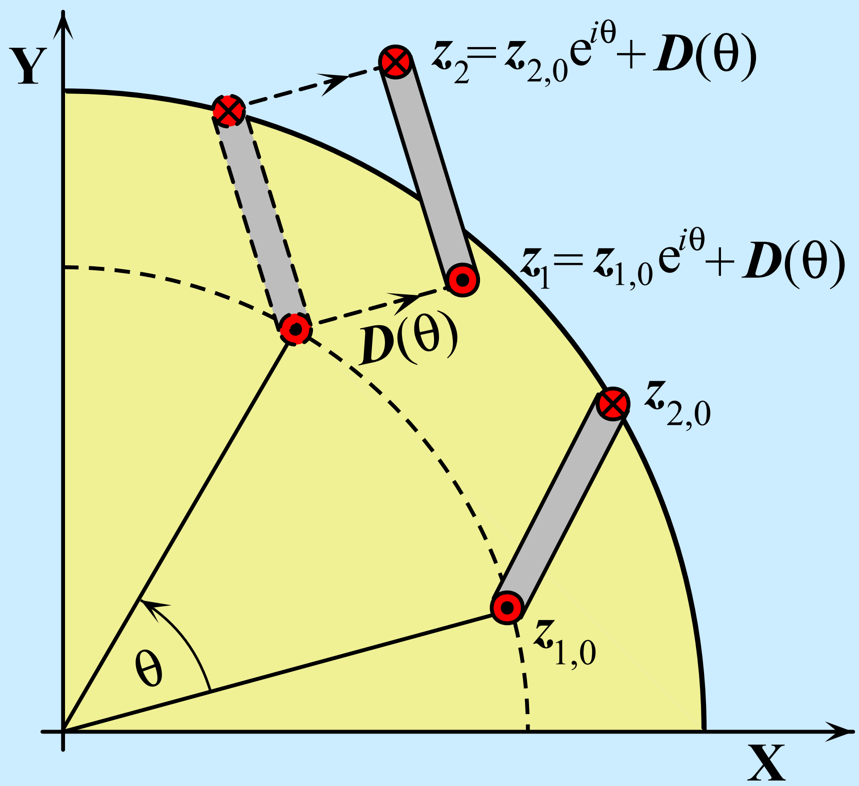
$$\boldsymbol{K}_n = \left(\frac{NLR_{ref}}{n}\right)\left\{\left(\frac{\boldsymbol{z}_{2,0}}{R_{ref}}\right)^n - \left(\frac{\boldsymbol{z}_{1,0}}{R_{ref}}\right)^n\right\}$$

The complex sensitivity factor gives both the amplitude and phase of the *n*-th harmonic term in the flux seen by the coil.

Imperfections in Coil Motion

- The rotation axis may move (wobble) by small amounts as the coil rotates.
- The coil angular position may not precisely match the expected angular position for a particular trigger. This can happen either due to torsional vibrations in the coil, or due to timing errors in the triggers.
- These imperfections produce spurious harmonics in the coil signal. These spurious harmonics can be minimized by employing ***"bucking"***.

Transverse Vibrations

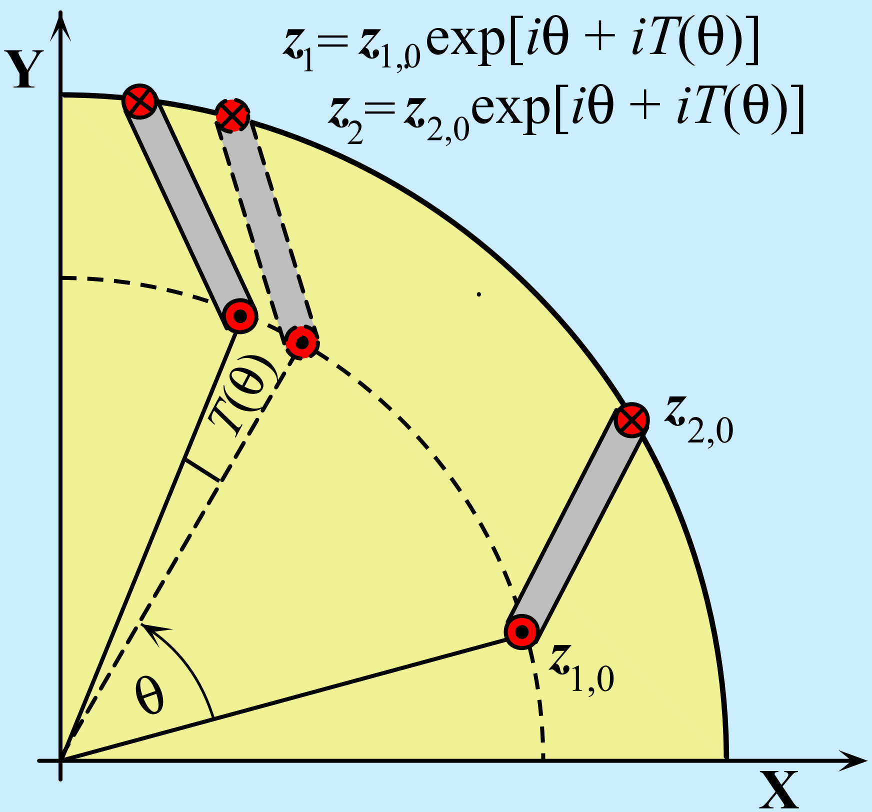


The coil is displaced from the ideal position by a vector $\boldsymbol{D}(\theta)$ when the coil rotates through θ.

In a pure 2*n*-pole field, the amount of spurious harmonics in the coil signal is roughly proportional to the sensitivity of the coil to the ($n-1$)th harmonic.

The effect of transverse vibrations in a 2*n*-pole magnet can be minimized by using a coil system whose sensitivity to the (*n*-1)th harmonic is zero.

Torsional Errors

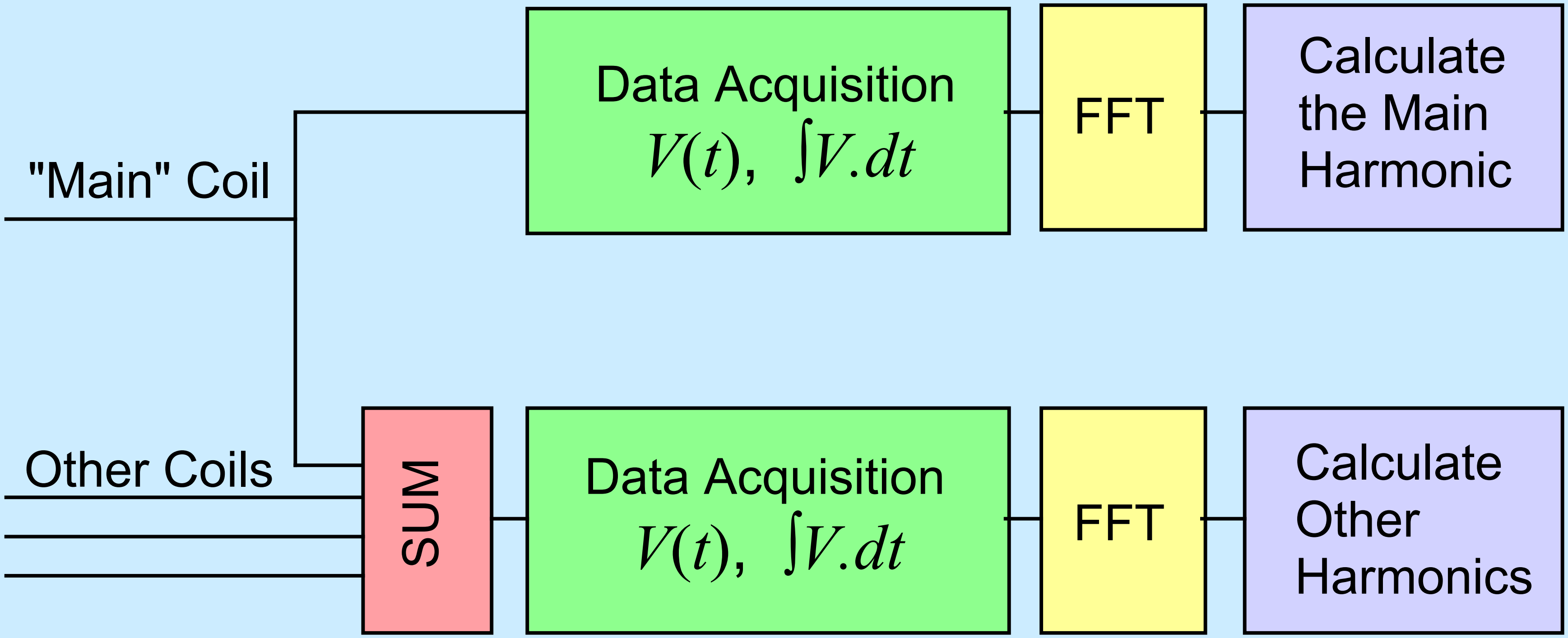


The coil angular position is $\theta + T(\theta)$ when it should have been θ.

In a pure 2*n*-pole field, the amount of spurious harmonics in the coil signal is roughly proportional to the sensitivity of the coil to the *n*-th harmonic.

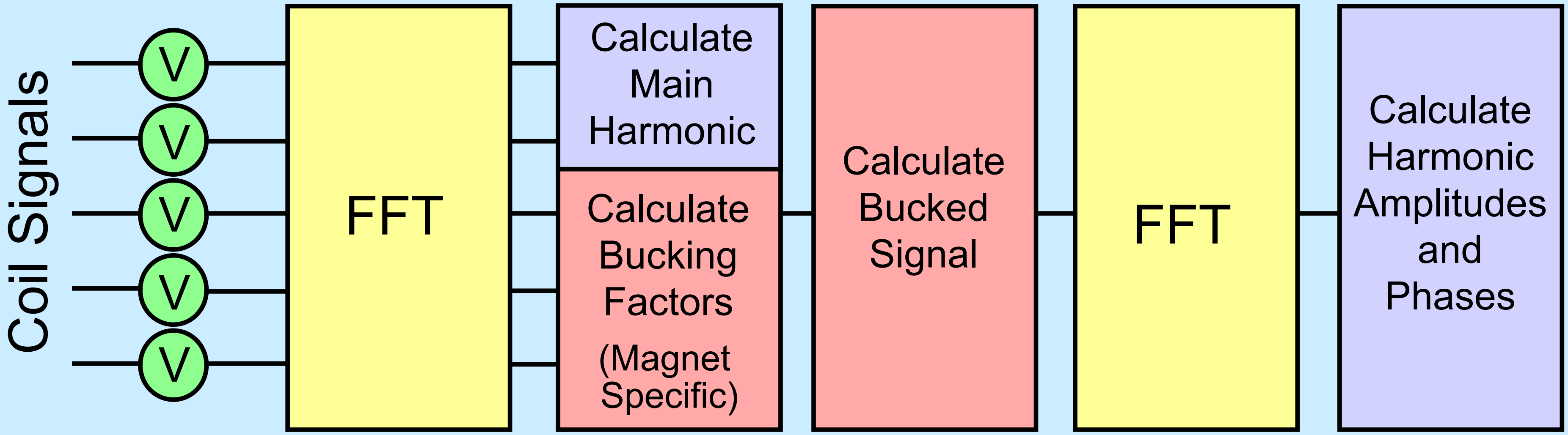
The effect of torsional errors in measuring a 2*n*-pole magnet can be minimized by using a coil system whose sensitivity to the *n*-th harmonic is zero.

Analog Bucking



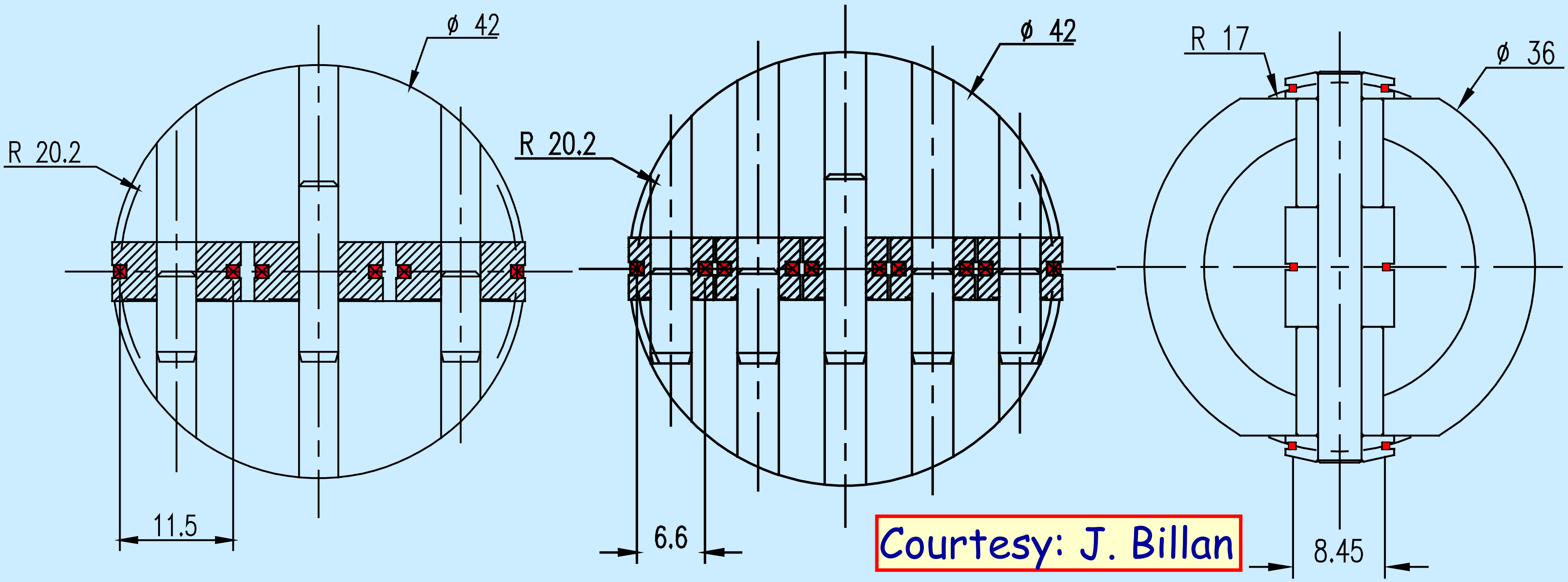
In Analog Bucking, one (or more) of the coils is used to measure the most dominant harmonic term (the "Main" term). The outputs of various coils are summed **BEFORE** recording the data.

Digital Bucking



In Digital Bucking, one (or more) of the coils is used to measure the most dominant harmonic term (the "Main" term). The direct outputs of various coils are also acquired and analyzed. The bucking is then carried out digitally, using factors determined from the measured harmonic contents in various coil signals.

Examples of Coil Designs: LHC

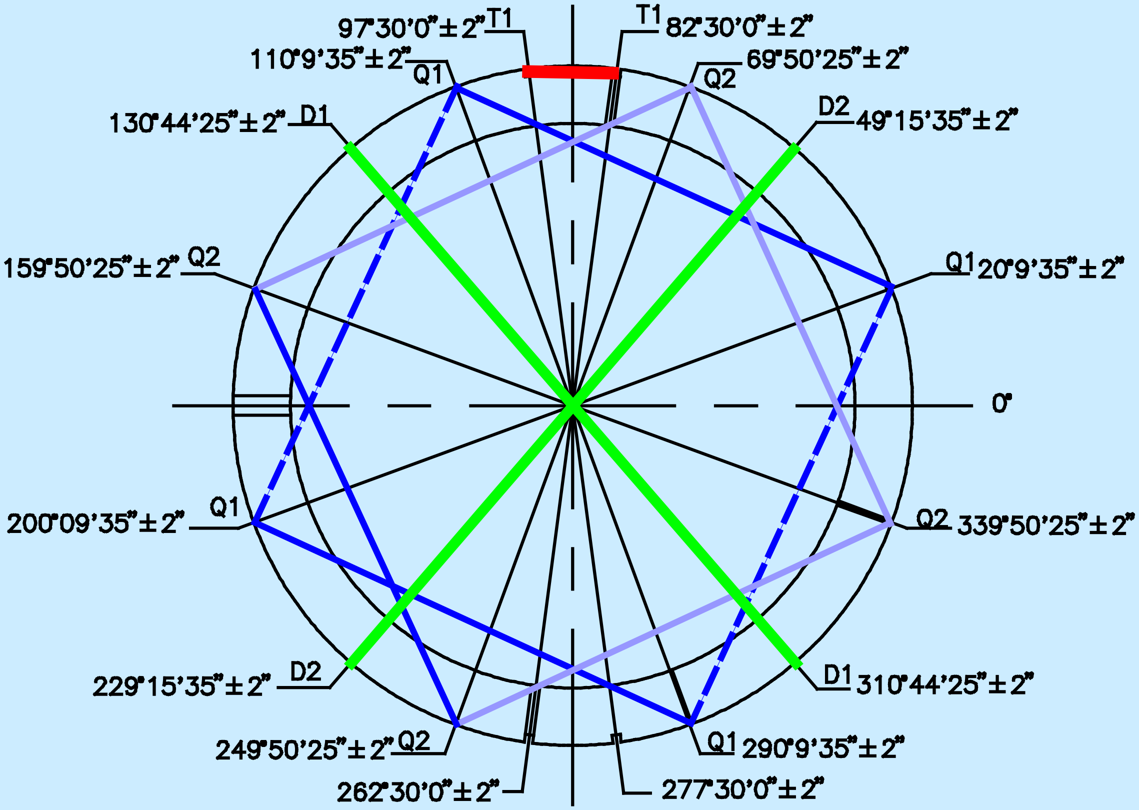


Dipole Coil (Radial) | **Quadrupole Coil (Radial)** | **Dipole Coil (Tangential)**

Only the dipole term needs to be "bucked" when measuring dipoles. Only one <u>well aligned</u> buck coil is sufficient.

For quadrupoles, both the dipole and the quadrupole terms must be cancelled. This requires multiple coils.

Examples of Coil Designs: RHIC



All RHIC coils were wound in grooves machined into G-10 cylinders. All coils had a similar cross section, with 5 active windings.

2 Dipole (3 turns each)
2 Quad (3 turns each)
1 Tangential (30 turns, 15^0)

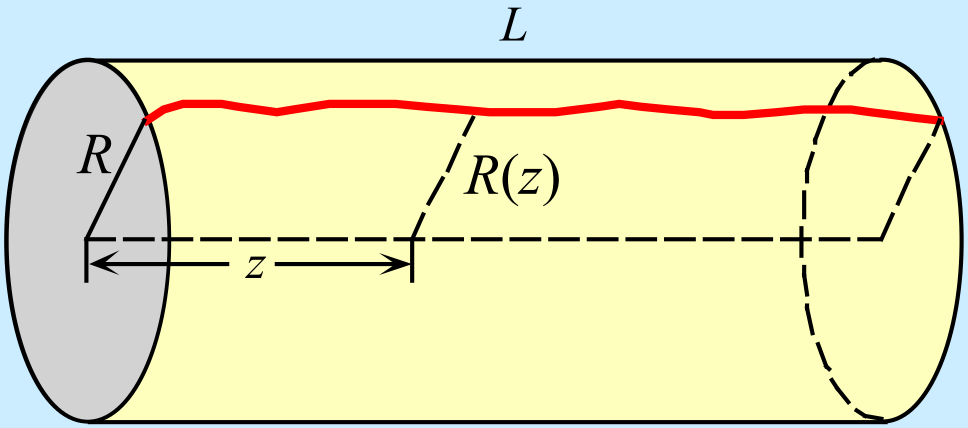
The same coil design can be used for measuring practically all types of magnets (Dipole through 12-pole, except Octupole) by automatically adjusting the weight factors in a digital bucking scheme.

Effects of Coil Construction Errors

- The construction of the rotating coil may have random and systematic errors, which may affect the measurement accuracy.
- Some of the errors (e.g. a systematic error in the coil radius) may be easily accounted for in the data analysis by using a suitable calibration.
- Many error types are difficult to calibrate and incorporate into the data analysis. It is important to understand the effect of such construction errors, and to keep such errors under tight control in the construction of the coil.

Examples of Construction Errors

Random variation in coil radius



$$R(z) = R_c + \varepsilon(z);\ \frac{1}{L}\int_0^L R(z)dz = R_c;$$

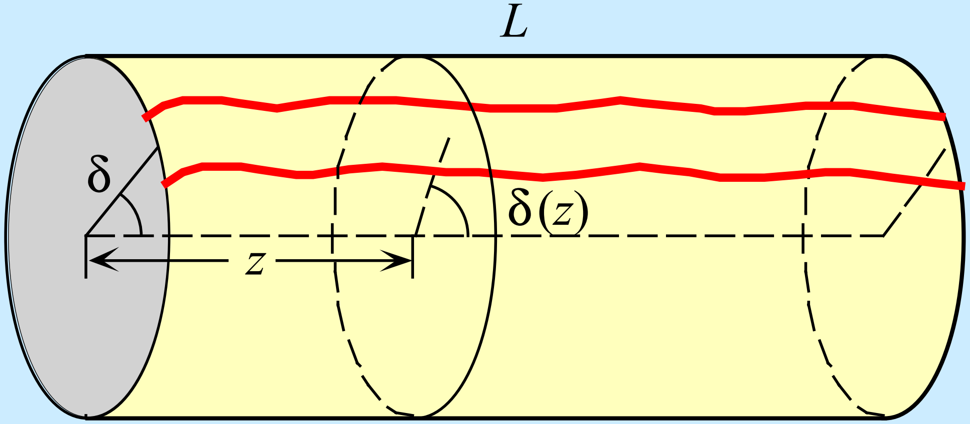
$$\int_0^L \varepsilon(z)dz = 0;\ \sigma_R^2 = \frac{1}{L}\int_0^L \left[R(z) - R_c\right]^2 dz$$

$$\frac{1}{L}\int_0^L \left[R(z)\right]^n dz = \frac{R_c^n}{L}\int_0^L \left[1 + \frac{n}{R_c}\varepsilon(z) + \frac{n(n-1)}{2R_c^2}\varepsilon^2(z) + \cdots\right] dz \approx R_c^n\left[1 + \frac{n(n-1)}{2}\left(\frac{\sigma_R}{R_c}\right)^2\right]$$

$$\boldsymbol{K}_n \approx \boldsymbol{K}_n^{ideal}\left[1 + \frac{n(n-1)}{2}\left(\frac{\sigma_R}{R_c}\right)^2\right]$$

Examples of Construction Errors

Random variation in coil angular position (twist)



$$\delta(z) = \delta_c + \varepsilon(z); \ \frac{1}{L}\int_0^L \delta(z)dz = \delta_c;$$

$$\int_0^L \varepsilon(z)dz = 0; \ \sigma_\delta^2 = \frac{1}{L}\int_0^L [\delta(z) - \delta_c]^2 dz$$

Flux from 2n-pole field: $\Phi_n(t) \propto \frac{1}{L}\int_0^L C(n)\sin(n\omega t + n\delta_c + n\varepsilon(z) - n\alpha_n)\,dz$

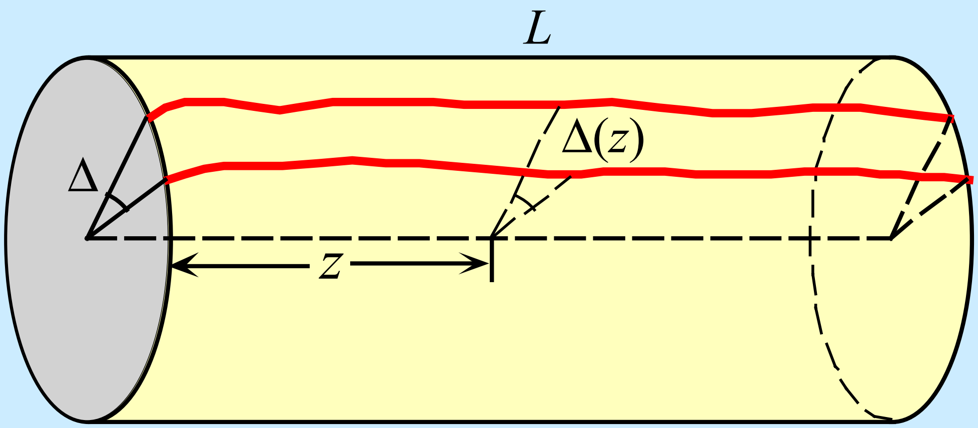
$$\Phi_n(t) \propto C(n)\sin(n\omega t + n\delta_c - n\alpha_n)\left[1 - \frac{n^2}{2}\sigma_\delta^2\right]$$

Expand sin[nε(z)] & cos[nε(z)] in power series

$$\boldsymbol{K}_n \approx \boldsymbol{K}_n^{ideal}\left[1 - \frac{n^2}{2}\sigma_\delta^2\right]$$

Examples of Construction Errors

Random variation in coil opening angle



$$\Delta(z) = \Delta_c + \varepsilon(z); \quad \frac{1}{L}\int_0^L \Delta(z)dz = \Delta_c$$

$$\int_0^L \varepsilon(z)dz = 0; \quad \sigma_\Delta^2 = \frac{1}{L}\int_0^L [\Delta(z) - \Delta_c]^2 dz$$

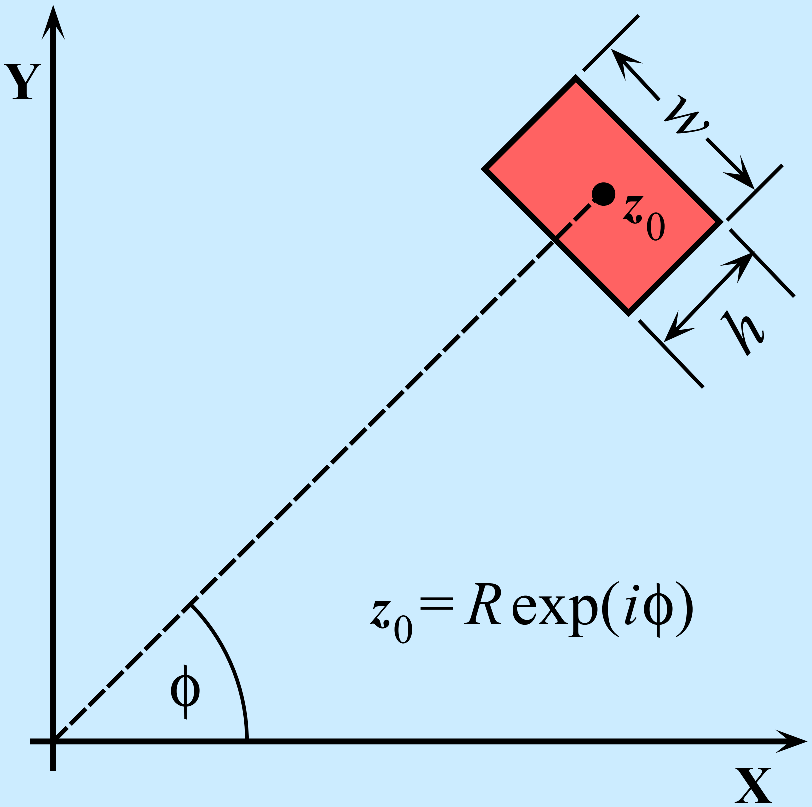
Flux from $2n$-pole field: $$\Phi_n(\theta) \propto \frac{1}{L}\int_0^L \sin\left(\frac{n\Delta(z)}{2}\right) dz$$

$$\Phi_n(\theta) \propto \sin\left(\frac{n\Delta_c}{2}\right)\left[1 - \frac{n^2}{8}\sigma_\Delta^2\right]$$

Expand sin[nε(z)/2] & cos[nε(z)/2] in power series

$$\boldsymbol{K}_n \approx \boldsymbol{K}_n^{ideal}\left[1 - \frac{n^2}{8}\sigma_\Delta^2\right]$$

Examples of Construction Errors



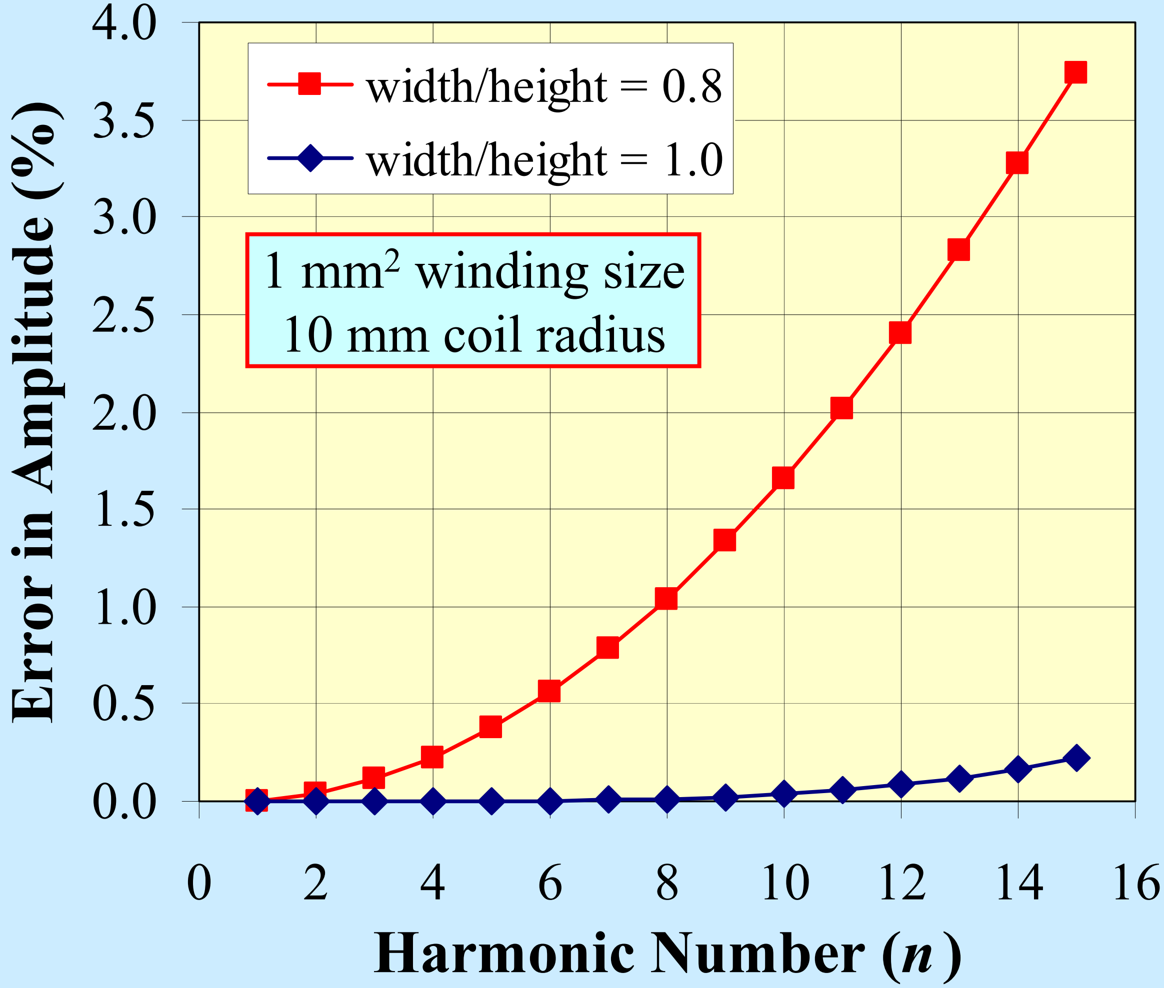
Finite Winding Size

$$\xi_{1,2} = \sqrt{\left(1 \pm \frac{h}{2R}\right)^2 + \left(\frac{w}{2R}\right)^2}$$

$$\lambda_{1,2} = \tan^{-1}\left[\frac{(w/2R)}{1 \pm (h/2R)}\right]$$

$$(z^n)_{avg.} = z_0^n \cdot \left[\frac{\xi_1^{n+2}\sin\{(n+2)\lambda_1\} - \xi_2^{n+2}\sin\{(n+2)\lambda_2\}}{2(h/2R)(w/2R)(n+1)(n+2)}\right]$$

Example: Finite Winding Size

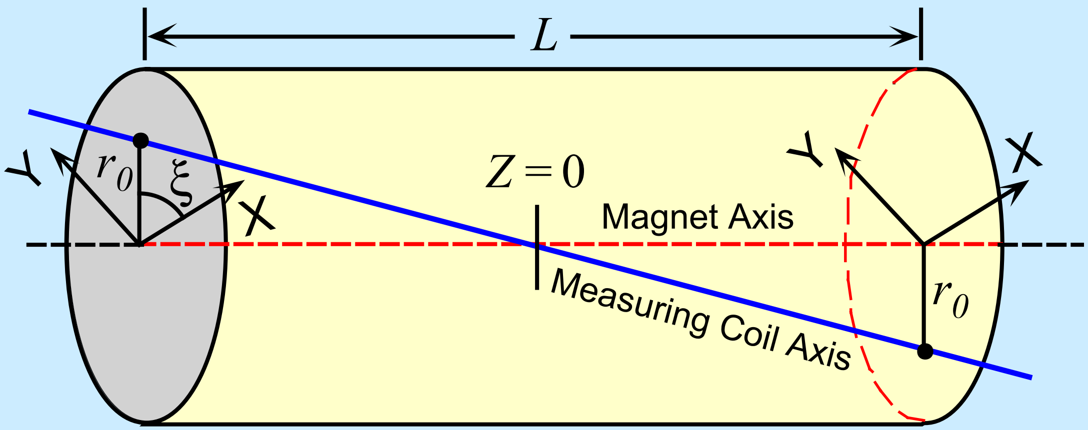


Errors due to a finite winding size are significantly less for a winding of square cross section. This is a result of cancellation of second order terms in (h/R) and (w/R).

Effects of Coil Placement Errors

- Even when the coil is perfectly built, and calibrated, errors may be introduced in the measurements simply by how the coil is placed in the magnet.
- If the coil rotation axis is not coincident with the magnetic axis, various harmonics will be mixed due to feed down. These errors can be corrected to a large extent by keeping the offsets to a minimum, and ***centering*** the data.
- The rotation axis of the coil may be ***tilted*** with respect to the magnetic axis. Errors due to this misalignment are difficult to correct, but affect only a very small class of measurements.

Tilt in the Measuring Coil Axis



If the field quality is uniform along the length, feed downs of odd orders over one half of the magnet cancel those over the other half.

$$B_n' + iA_n' = \sum_{\substack{k=n \\ (k-n)=\text{even}}}^{\infty} \frac{B_k + iA_k}{(k-n+1)} \frac{(k-1)!}{(n-1)!(k-n)!} \left(\frac{r_0 \exp(i\xi)}{R_{ref}} \right)^{k-n}$$

This error can be neglected if the main harmonic term can not have a second order feed down term, e.g. in measuring dipole and quadrupole magnets. However, for higher multipole magnets, the main harmonic can have a very strong second order feed down term, and cause significant errors (e.g. dipole term in sextupoles).

A tilt can also cause large (1^{st} order) errors if the field is not uniform along the axis.

Calibration of Rotating Coils

- For accurate work, good calibration of the coil is as important as precise fabrication.
- Generally, most parameters of interest can be obtained by carrying out measurements in ***known*** (strength and angle) dipole and quadrupole fields.
- For a coil of radius ~20 mm, it is possible to attain an absolute accuracy of ~0.02% for the main field in dipoles.
- With good calibration, and data analysis, errors in higher harmonics (at the coil radius) are below 10 ppm of the main field.

Summary

- Rotating coils provide the most convenient and accurate means of measuring field harmonics in typical accelerator magnets.
- Two geometries are commonly used – ***Radial*** and ***Tangential*** coils.
- Special geometries, such as ***multipole*** coils, are also used for specific harmonics.
- The signals may be recorded using either ***voltmeters*** or ***integrators***.

Summary (Contd.)

- Imperfections in coil *rotation* are compensated by use of ***bucking*** – either ***digital***, or ***analog***.
- Coil ***placement errors*** can either be corrected (***centering***) or are of significance only in some special cases (***tilt***).
- Imperfections in coil ***construction*** should be kept to a minimum.
- Good ***calibration*** is equally important!

Some References for More Information

- L. Walckiers, *The Harmonic Coil Method*, CERN Accelerator School on Magnetic Measurement and Alignment, Montreux, Switzerland, March 16-20, 1992; CERN Report 92-05, pp. 138-166. (http://preprints.cern.ch/cernrep/1992/92-05/92-05.html)
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- A. K. Jain, *Harmonic Coils*, CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets, Anacapri, Italy, April 11-17, 1997; CERN Report 98-05, pp. 175-217. (http://preprints.cern.ch/cernrep/1998/98-05/98-05.html)
- M. I. Green, *Search Coils*, CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets, Anacapri, Italy, April 11-17, 1997; CERN Report 98-05, pp. 143-173.(http://preprints.cern.ch/cernrep/1998/98-05/98-05.html)