Determination of Magnetic Axis

Animesh Jain
Brookhaven National Laboratory
Upton, New York 11973-5000, USA

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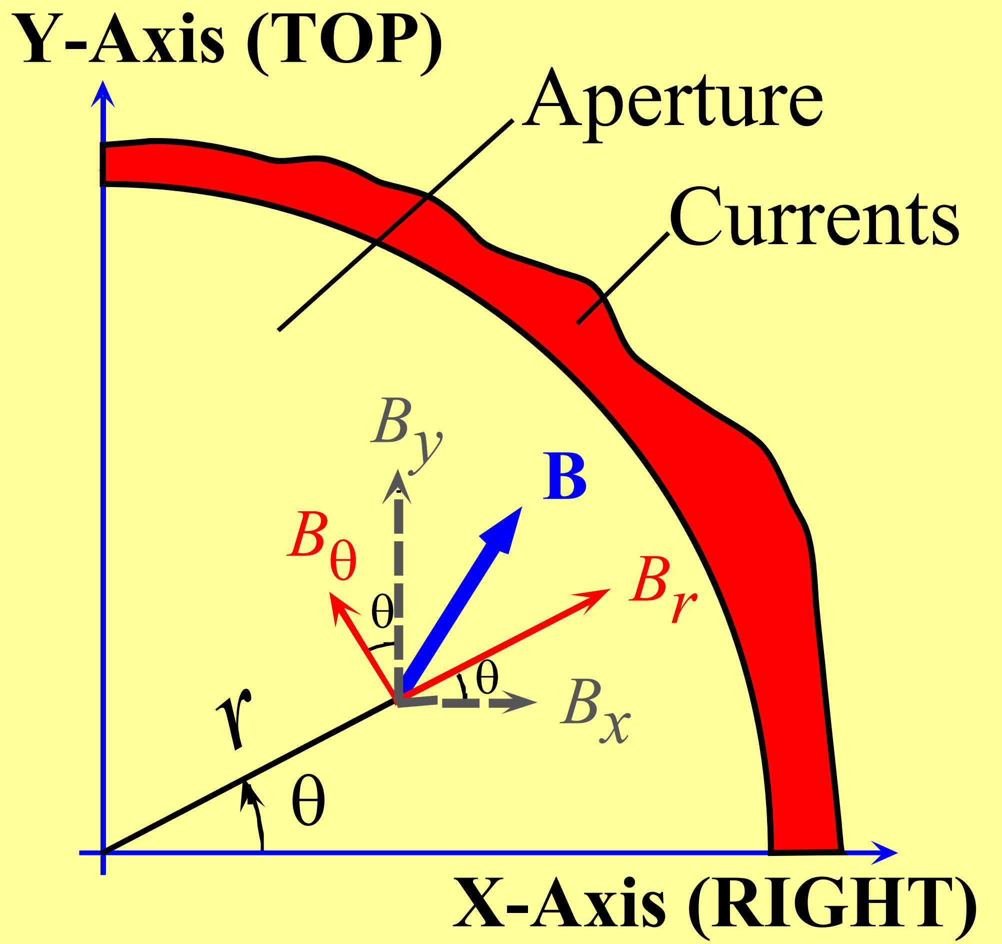
Introduction

- A misalignment of beam and the magnetic axis has unwanted effects on the beam due to feed down.
- For installing magnets in an accelerator, it is necessary to ensure that the magnetic axis coincides with the nominal beam axis.
- To achieve this goal, it is necessary not only to accurately ***locate the magnetic axis***, but also ***relate it to external magnet fiducials***.

Useful Background Information

Coordinate System for Describing Field



View from the Lead End of the Magnet

Useful Background Information

Expressions for the 2-D Field Components

$$B_r(r,\theta) = \sum_{n=n_0}^{\infty}\left[B_n \sin\{(n-n_0+1)\theta\} + A_n \cos\{(n-n_0+1)\theta\}\right]\left(\frac{r}{R_{ref}}\right)^{n-n_0}$$

$$B_\theta(r,\theta) = \sum_{n=n_0}^{\infty}\left[B_n \cos\{(n-n_0+1)\theta\} - A_n \sin\{(n-n_0+1)\theta\}\right]\left(\frac{r}{R_{ref}}\right)^{n-n_0}$$

(n_0=1 followed in this talk)

$n_0 = 0$: US

$n_0 = 1$: European

R_{ref}= Reference radius

$$B_y + iB_x = \sum_{n=n_0}^{\infty}\left[B_n + iA_n\right]\left(\frac{x+iy}{R_{ref}}\right)^{n-n_0}$$

B_{n+n_0-1} = $2n$ - pole NORMAL Term

A_{n+n_0-1} = $2n$ - pole SKEW Term

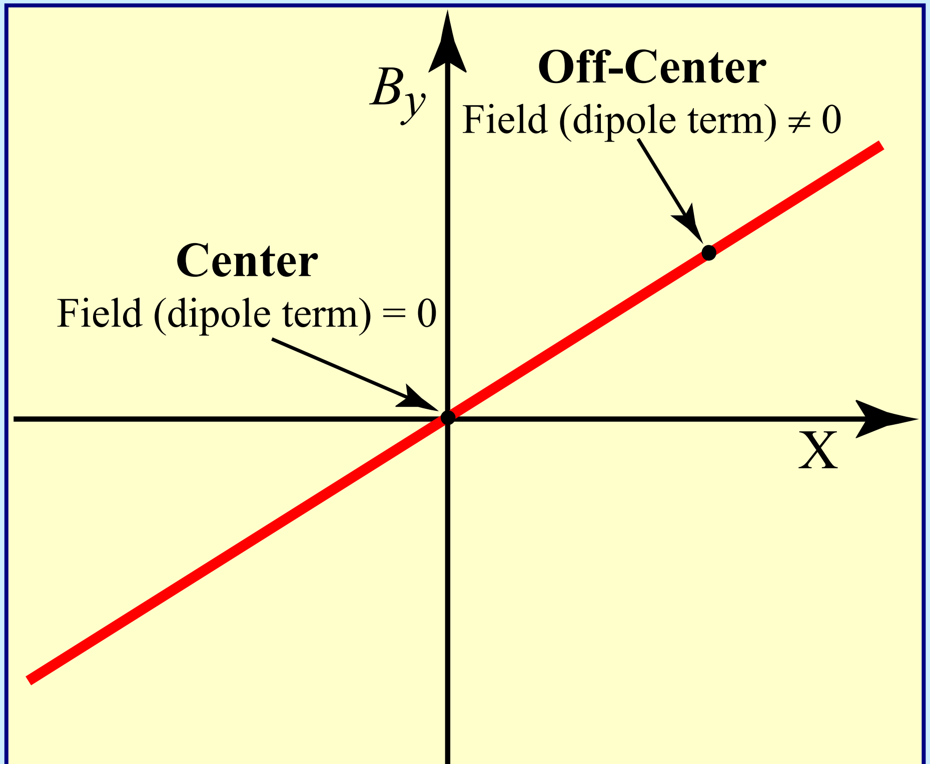
For $n > n_0$

$|B| = 0$ for $r = 0$

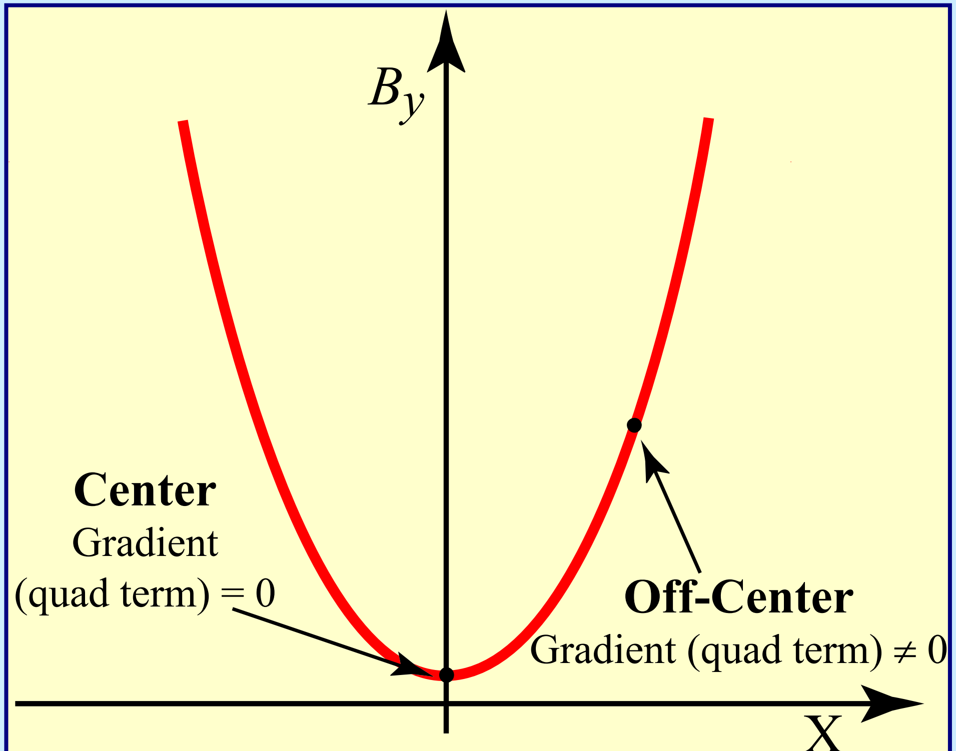
Definition of Magnetic Axis

- **For all 2*m*-pole magnets except dipoles, the magnetic axis is defined as the locus of points along which the 2(*m*–1) pole terms are zero.**
- In general, the axis, as defined above, follows an irregular path. However, the location of the axis in an ***integral*** sense is often of the most interest.
- Several techniques exist for both local and integral measurements of the magnetic axis.

Illustration of Feed Down



Field Profile in a Normal Quadrupole



Field Profile in a Normal Sextupole

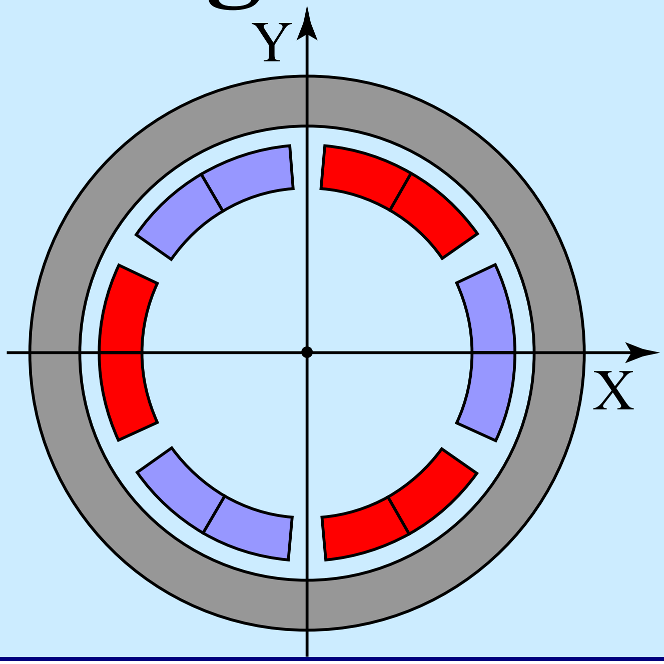
A dipole error term in a quadrupole is the same as a shift in magnetic center.

This is a sextupole with a dipole error term. A *pure* sextupole has zero field at the center.

Determination of Magnetic Axis

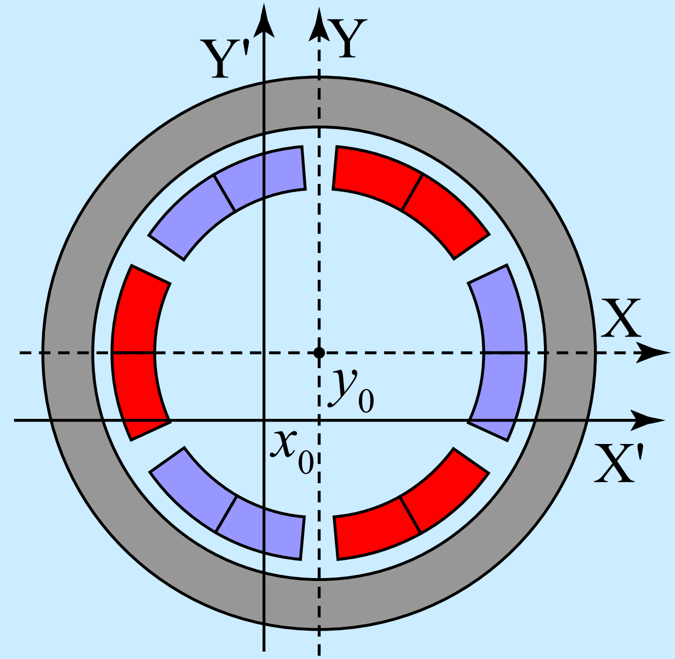
- If all the coefficients in the ***harmonic expansion*** of the field are measured in a known reference frame (e.g. using rotating coils), the magnetic center can be derived.
- The magnetic axis from harmonics can be integral or local, based on the probe length.
- Other techniques, such as the ***stretched wire***, or a ***colloidal cell***, exploit the ***symmetry conditions*** of the field around the magnetic center.

Magnetic Axis from Harmonics



Pure 2*m*-pole Field
(Reference Frame Aligned)

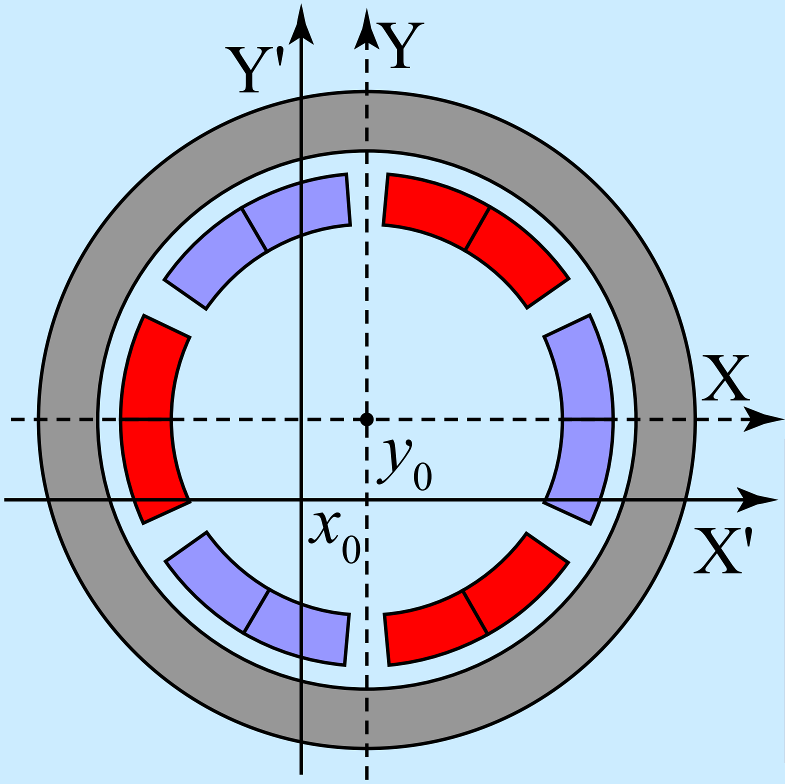
$$B_y + iB_x = (B_m + iA_m)\left(\frac{x+iy}{R_{ref}}\right)^{m-1}$$



Pure 2*m*-pole Field
(Reference Frame Offset)

$$B_{y'} + iB_{x'} = B_y + iB_x = (B_m + iA_m)\left[\frac{(x'-x_0)+i(y'-y_0)}{R_{ref}}\right]^{m-1}$$

Magnetic Axis from Harmonics



Pure $2m$-pole Field
(Reference Frame Offset)

$$B_{y'} + iB_{x'} = B_y + iB_x$$

$$= (B_m + iA_m)\left[\frac{(x' - x_0) + i(y' - y_0)}{R_{ref}}\right]^{m-1}$$

Harmonics in the offset frame are:

$$B'_m + iA'_m = B_m + iA_m$$

$$B'_{m-1} + iA'_{m-1} = (m-1)(B_m + iA_m)\left[\frac{x_0 + iy_0}{R_{ref}}\right]$$

Location of Magnetic Axis is:

$$x_0 + iy_0 = \left(\frac{1}{m-1}\right)\left(\frac{B'_{m-1} + iA'_{m-1}}{B'_m + iA'_m}\right) R_{ref}$$

Magnetic Axis of a Dipole

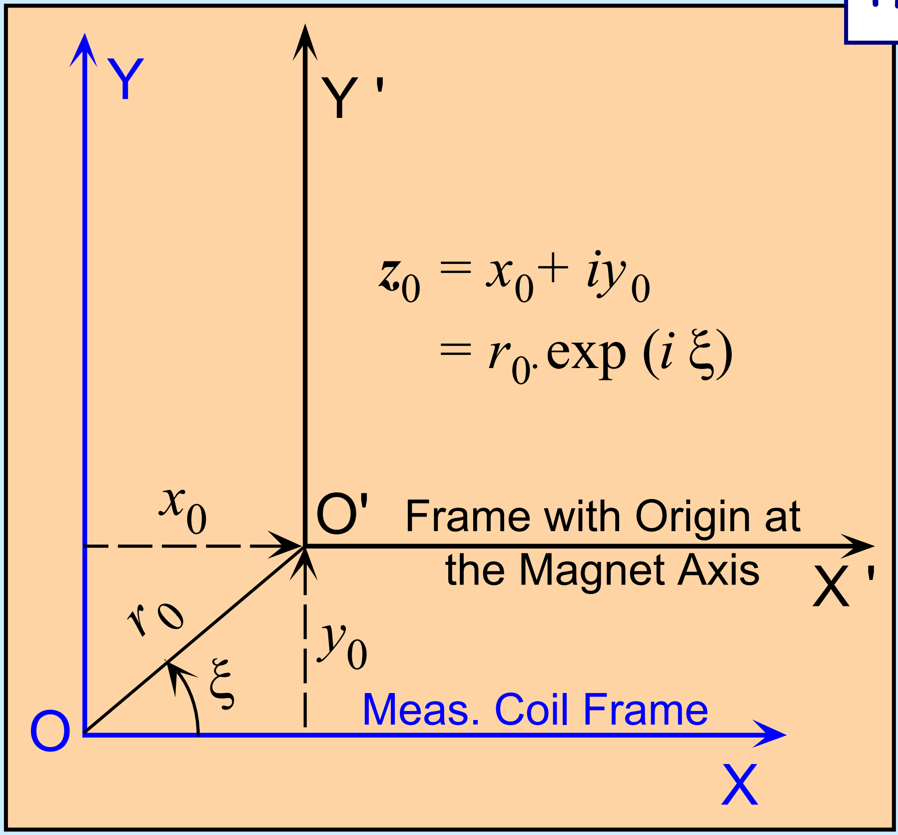
- A *pure* dipole field is uniform across the entire aperture, and thus has no well defined axis.
- In reality, a dipole magnet has non-zero higher harmonics, either by design, or due to construction errors. It may be necessary to worry about feed down from such terms.
- For a *perfectly built* dipole, the magnetic axis may be defined as the axis of symmetry of the magnet coils. However, this definition is not very useful in practice.

Dipole Axis from Higher Harmonics

- Practical dipole designs often have small, but finite *allowed* harmonics of very high order (18-pole and above) due to design limitations.
- The higher order *unallowed* harmonics are expected to be nearly zero at a radius of interest.
- One could define the magnetic axis of the dipole as the axis where a suitable high order *unallowed* term is zero.
- For this to work well, the magnet design should have a healthy amount (~0.5 unit or more) of high order *allowed* term.

Feed Down from Higher Harmonics

If $(m-1)$ is an unallowed harmonic, its value in the magnet frame is:



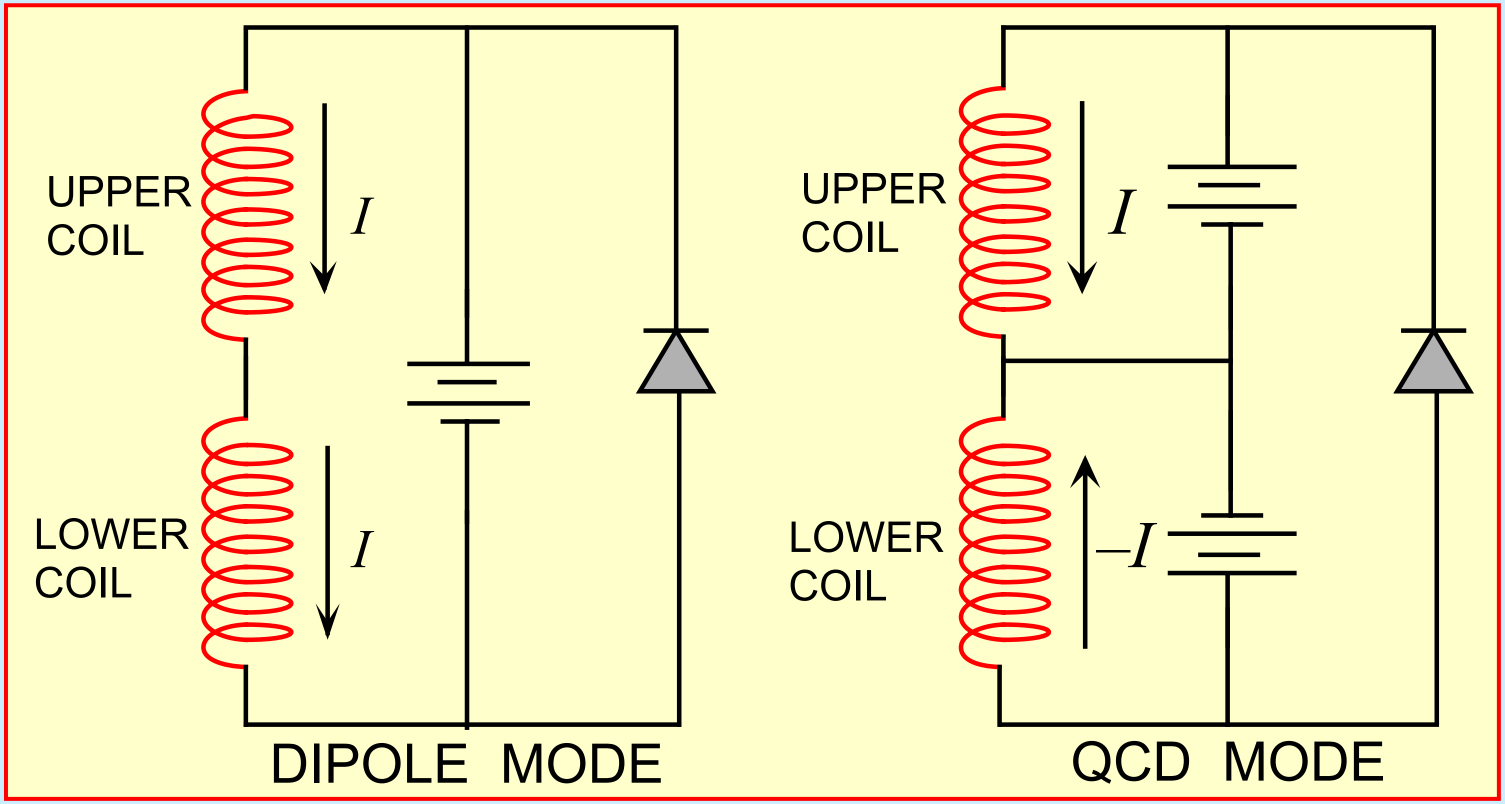
$$
\begin{aligned}
B'_{m-1} + iA'_{m-1} &= B_{m-1} + iA_{m-1} \\
&+ (m-1)\left(B_m + iA_m\right)\left(\frac{z_0}{R_{ref}}\right) \\
&+ \frac{m(m-1)}{2}\left(B_{m+1} + iA_{m+1}\right)\left(\frac{z_0}{R_{ref}}\right)^2 \\
&+ \frac{m(m^2-1)}{6}\left(B_{m+2} + iA_{m+2}\right)\left(\frac{z_0}{R_{ref}}\right)^3 + \cdots \\
&= 0
\end{aligned}
$$

- The offset can be determined from the measured harmonics.
- It may be necessary to include higher order feed down terms.
- **Non-linear expression may result in multiple solutions.**

Dipole Axis: Another Option

- One could power the two coils in a dipole with opposite polarity of current.
- This produces a strong skew quadrupole field, instead of a dipole field, whose center can be defined with ease. (Top-bottom & Left-right Antisymmetry)
- This also produces fairly strong skew octupole, dodecapole, etc. terms, which could also be used to determine the magnetic axis.
- This method, known as the ***quadrupole configured dipole*** method, was first proposed at SSC, and was used in all the arc dipoles for RHIC.

Quadrupole Configured Dipole

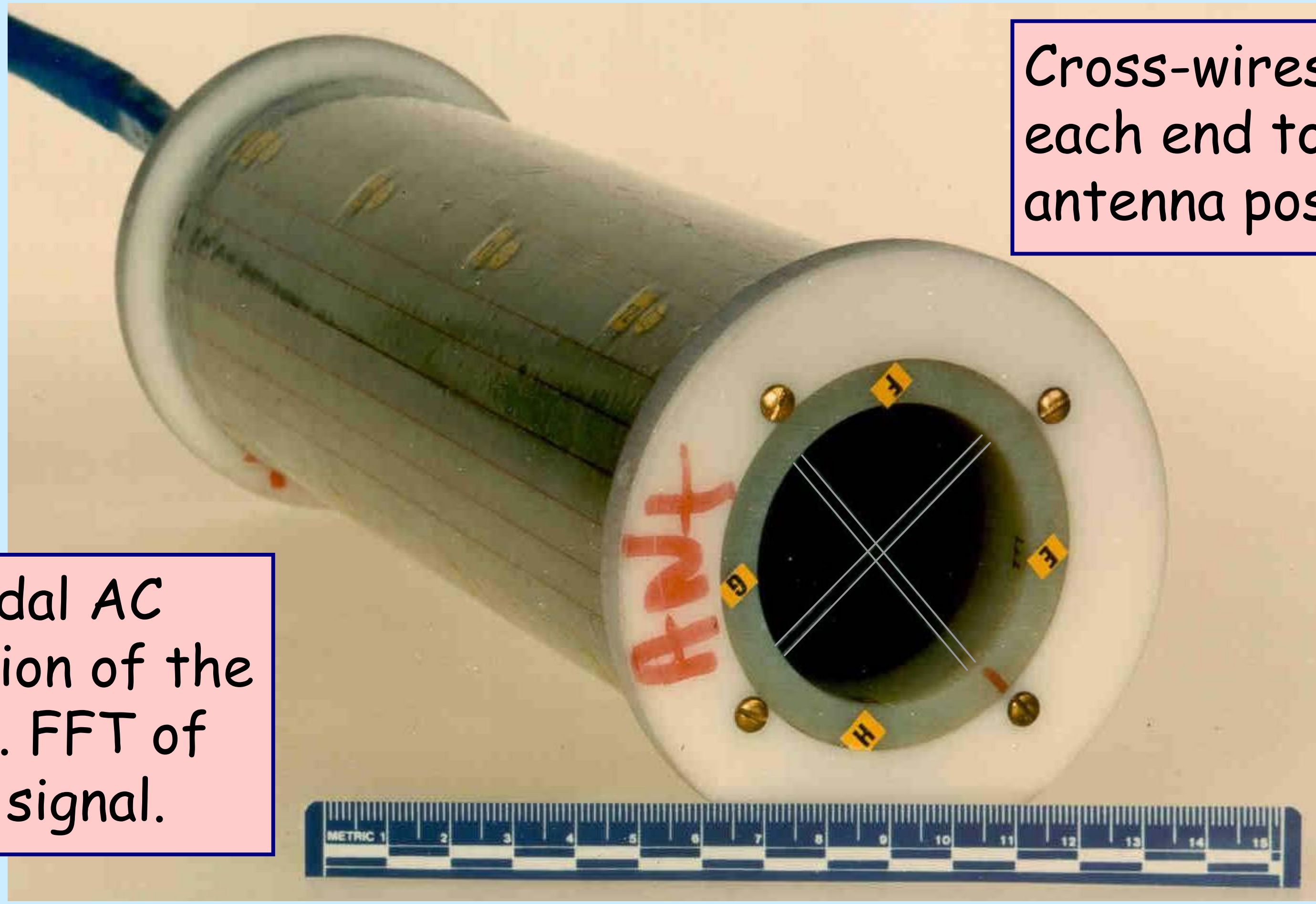


In the Quadrupole Configured Dipole mode, one needs two power supplies with precisely matched currents. A mismatch in current produces a dipole field in addition to the skew quadrupole, thus causing an error in the axis determination. This error is greatly reduced if higher order terms are used, instead of skew quad.

Magnetic Axis from Harmonics

- One can measure all harmonic coefficients quite easily using a rotating coil.
- Since only two harmonics are of prime importance in determining the axis, one can also use a non-rotating coil with special windings. This method requires powering the magnet with an AC current.
- In either method, one also needs a means of locating the axis of the measuring system relative to outside fiducials. This is relatively easy for a non-rotating system.

Non-rotating Harmonic Coil (Antenna)

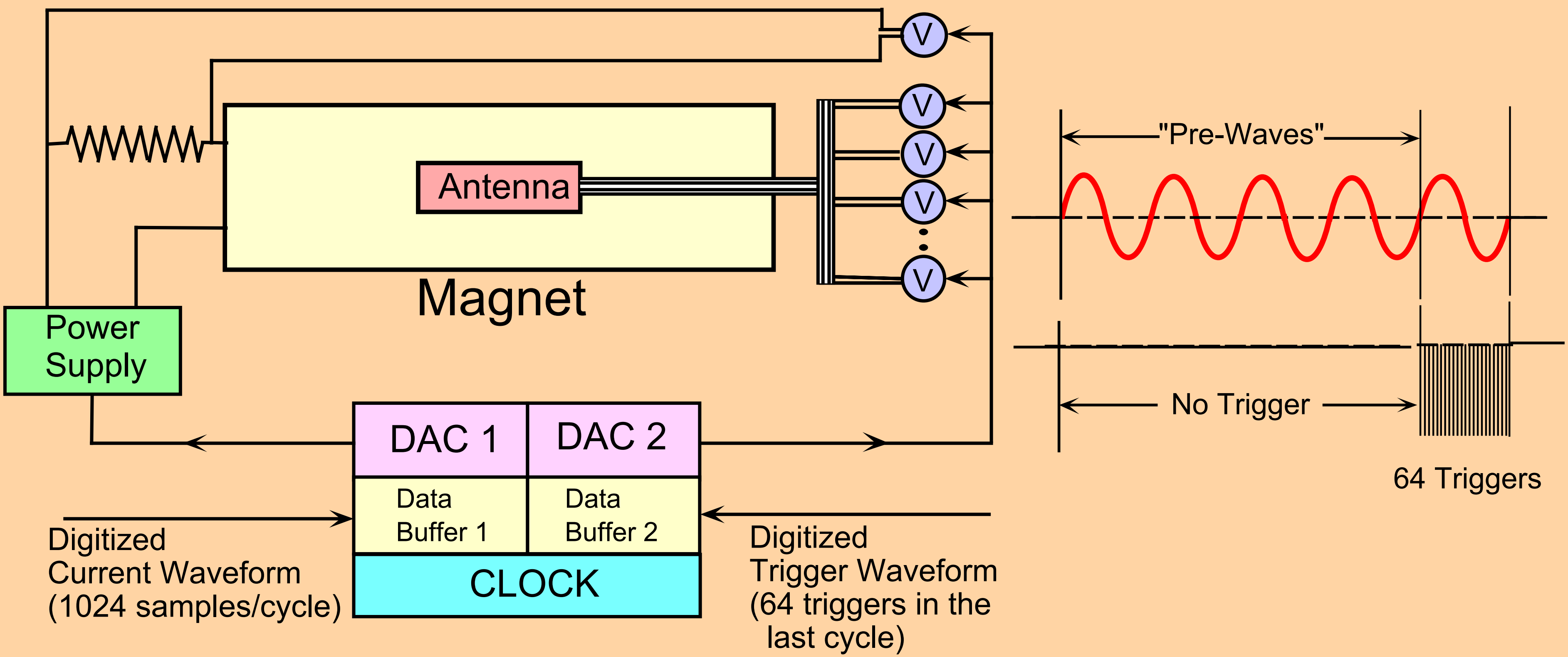


Cross-wires on each end to locate antenna position

Sinusoidal AC excitation of the magnet. FFT of pick up signal.

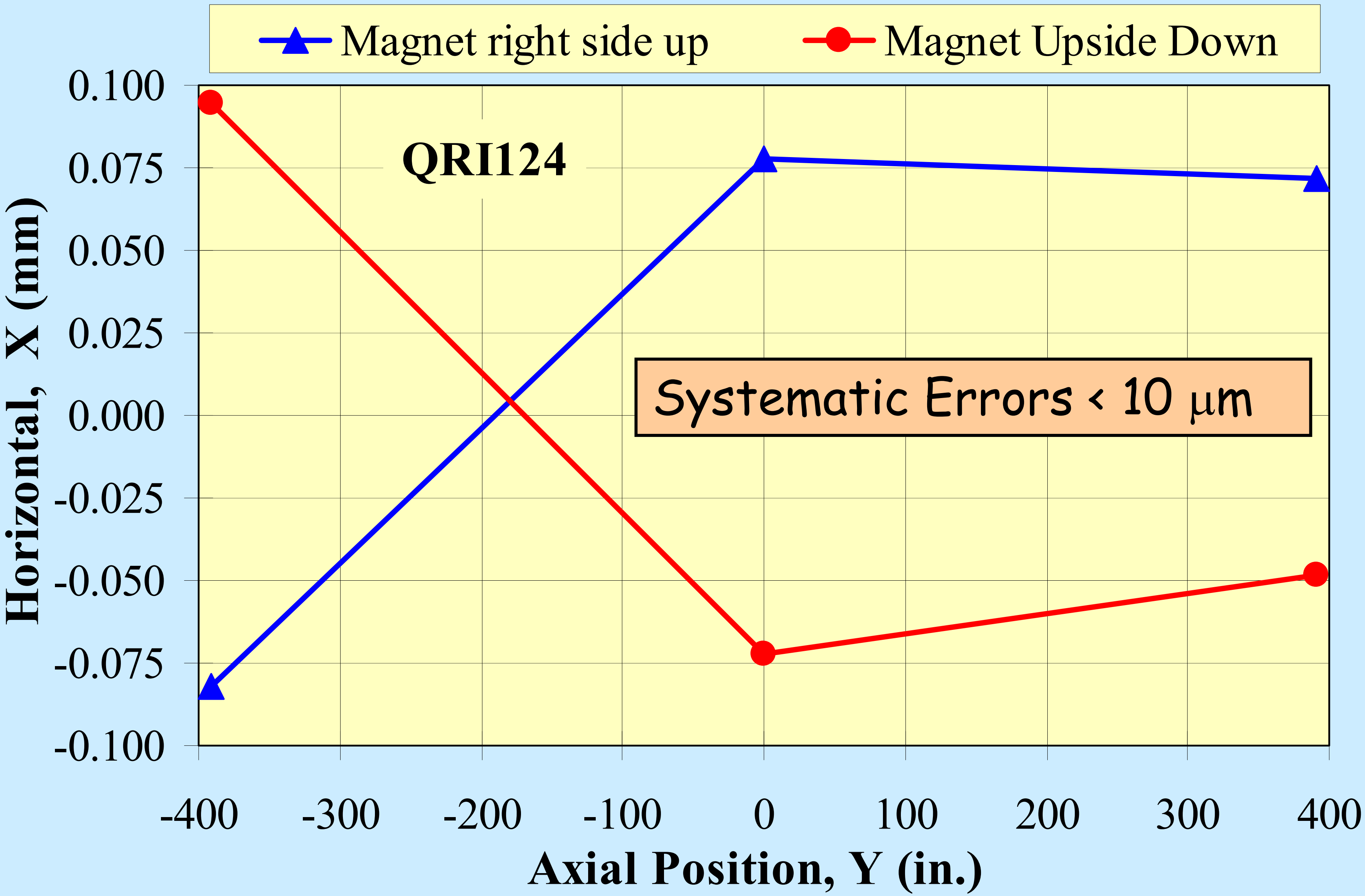
6 Windings (2 Dipole; 2 Quadrupole; 2 Octupole)
Can measure Quadrupoles through 12-pole magnets

Data Acquisition: Harmonic Antenna

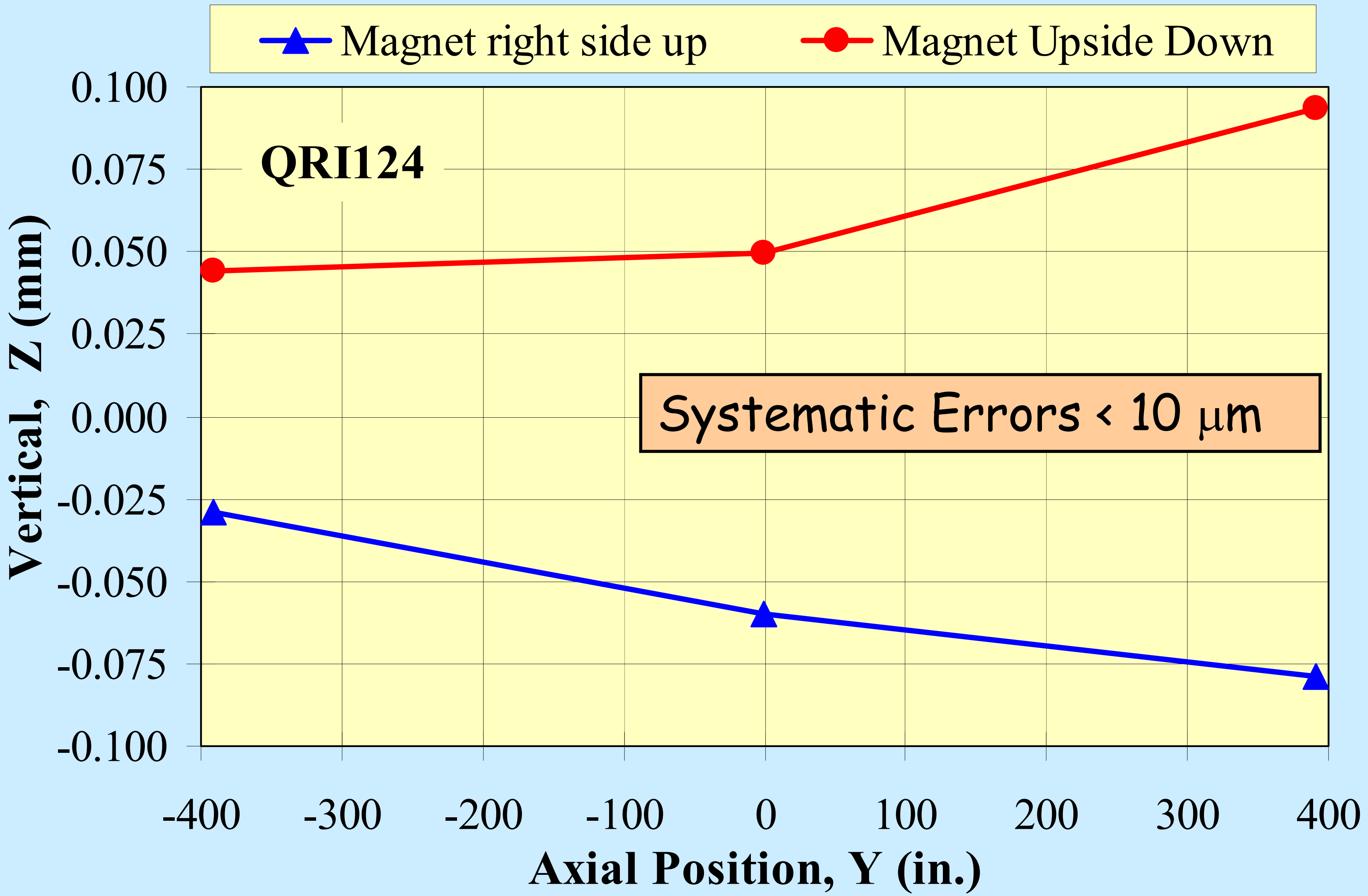


Details of data analysis can be found in A. Jain et al., IMMW-10, Fermilab, 1997.

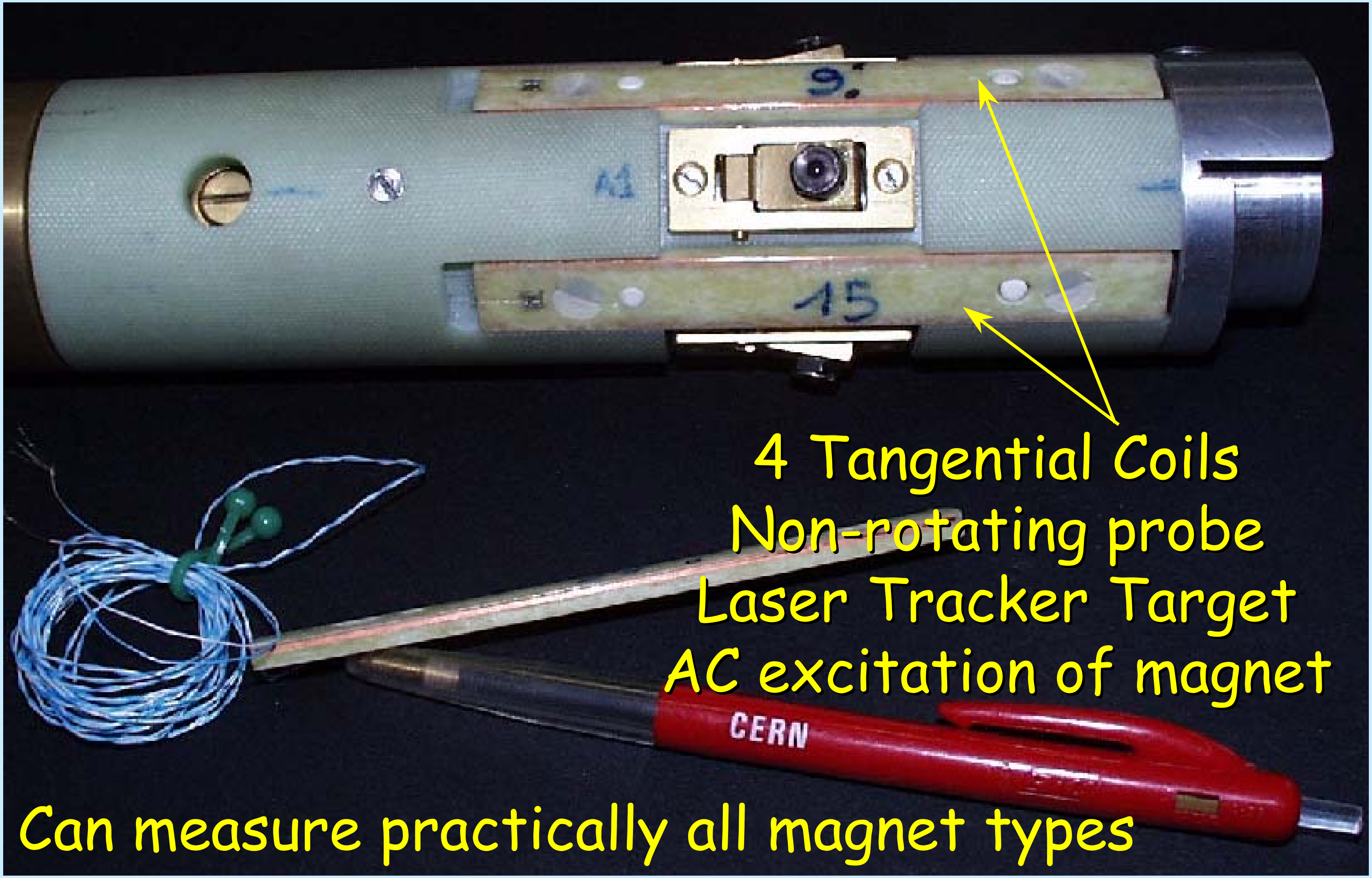
Harmonic Antenna Systematic Errors



Harmonic Antenna Systematic Errors



"Global Mole" for LHC

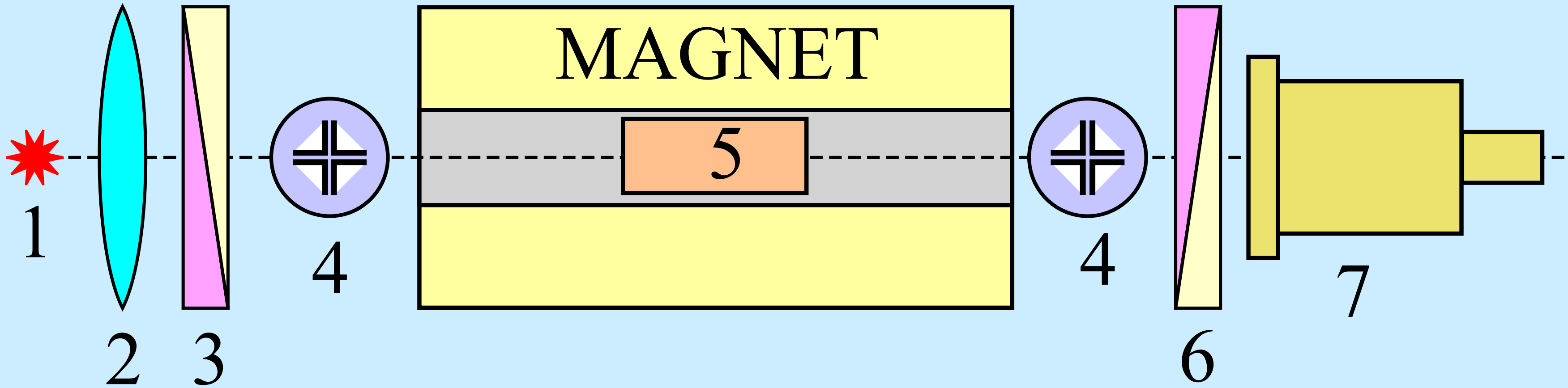


from: J. Billan & J. Garcia Perez, IMMW-XI, 1999

Magnetic Axis Using Field Symmetry

- Instead of measuring harmonics, one can simply try to probe the symmetry of the field near the axis of the magnet.
- A direct way of probing this symmetry is by using a ***colloidal solution*** of ferromagnetic particles. These particles orient themselves along the field lines, and the pattern can be visualized using polarized light.
- This method can measure the axis locally (average over ~ a few centimeters length)

The Colloidal Cell Technique

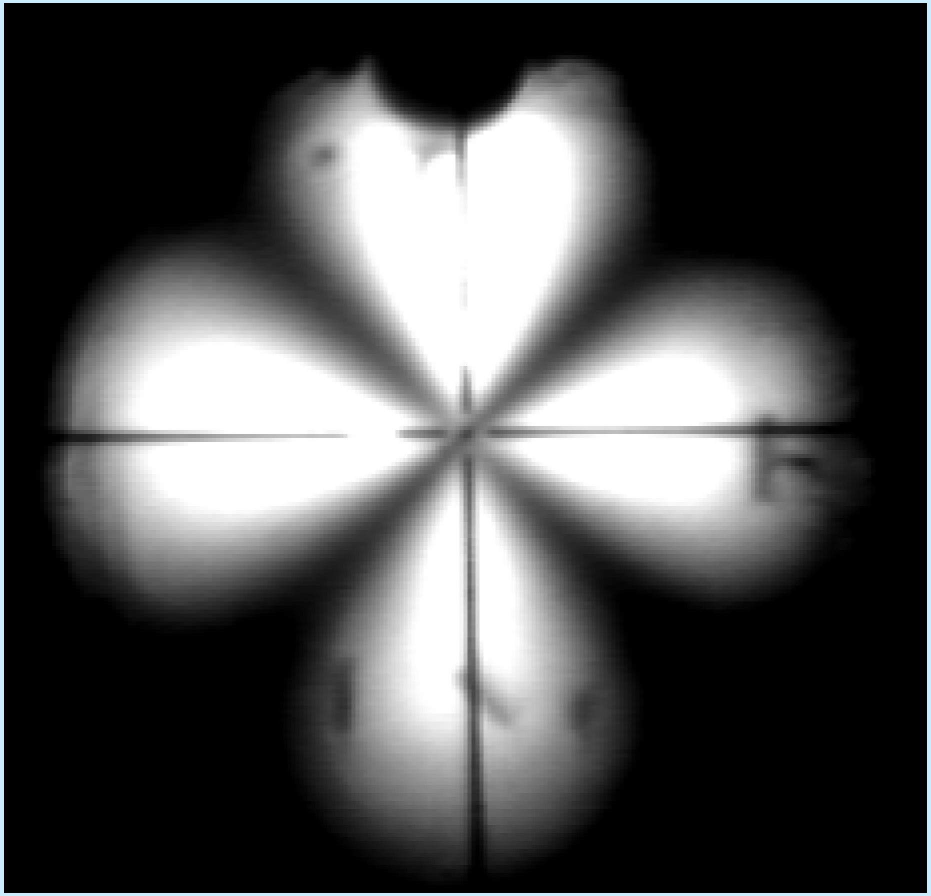


Schematic of Colloidal Cell Set up

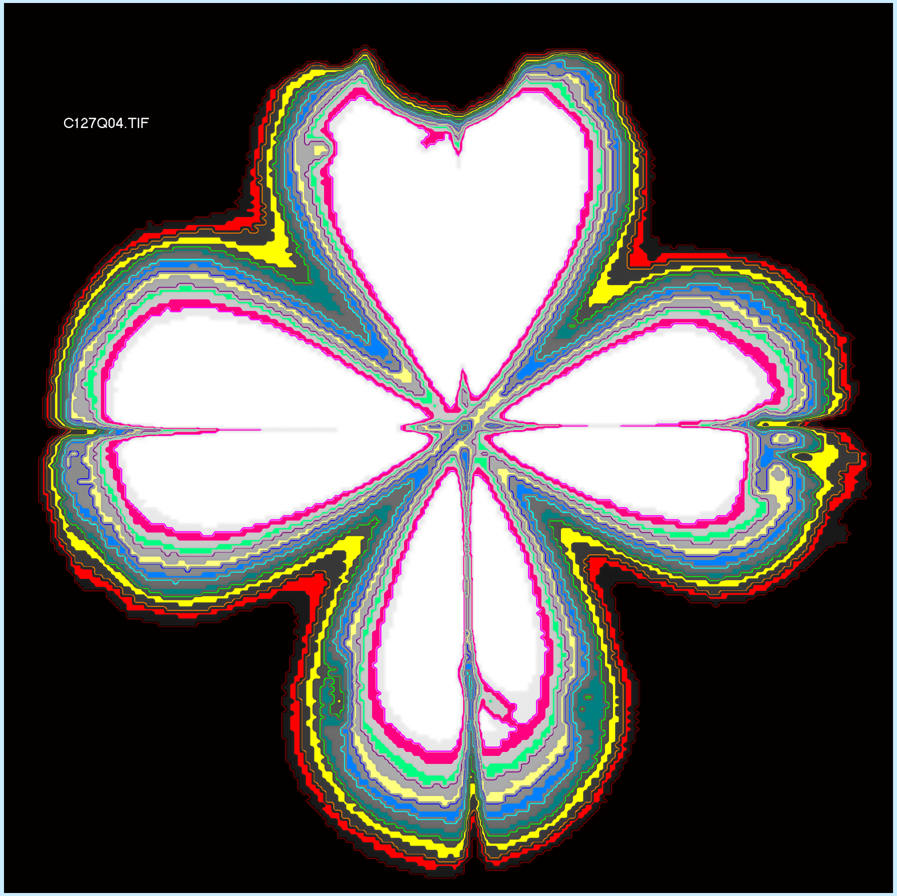
1= Light Source; 2= Collimator; 3= Polarizer;
4= Taylor-Hobson Balls; 5= Colloidal Cell;
6= Analyzer; 7= Alignment Telescope & Camera

A pair of Taylor-Hobson balls provides the reference axis for measurements. These balls are removed to view the pattern in the colloidal cell. The polarizer and the analyzer are set at 90 degrees to each other.

Colloidal Cell Pattern in Quadrupole

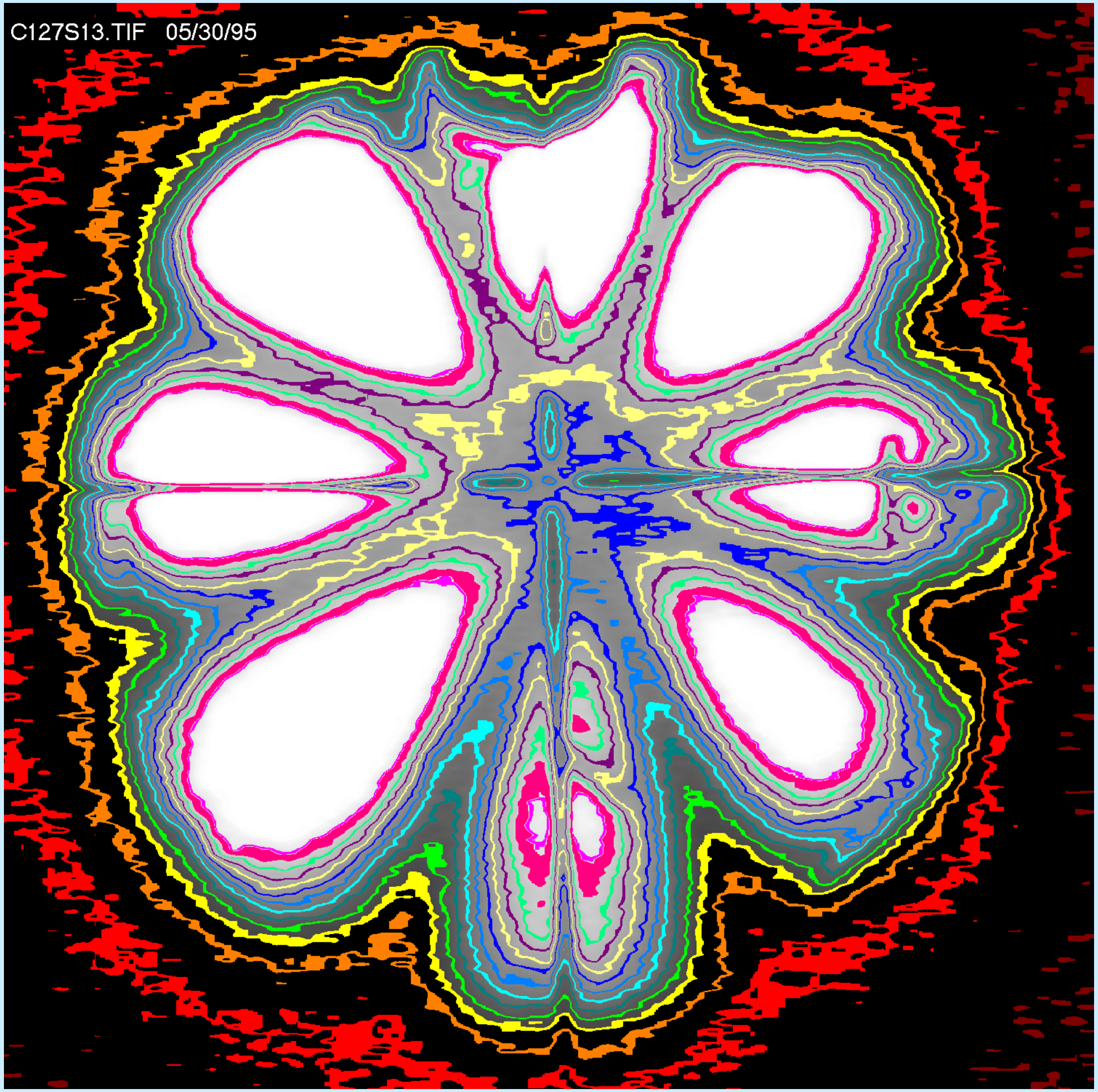


As recorded by CCD Camera



"Colorized" by software

Colloidal Cell Pattern in Sextupole



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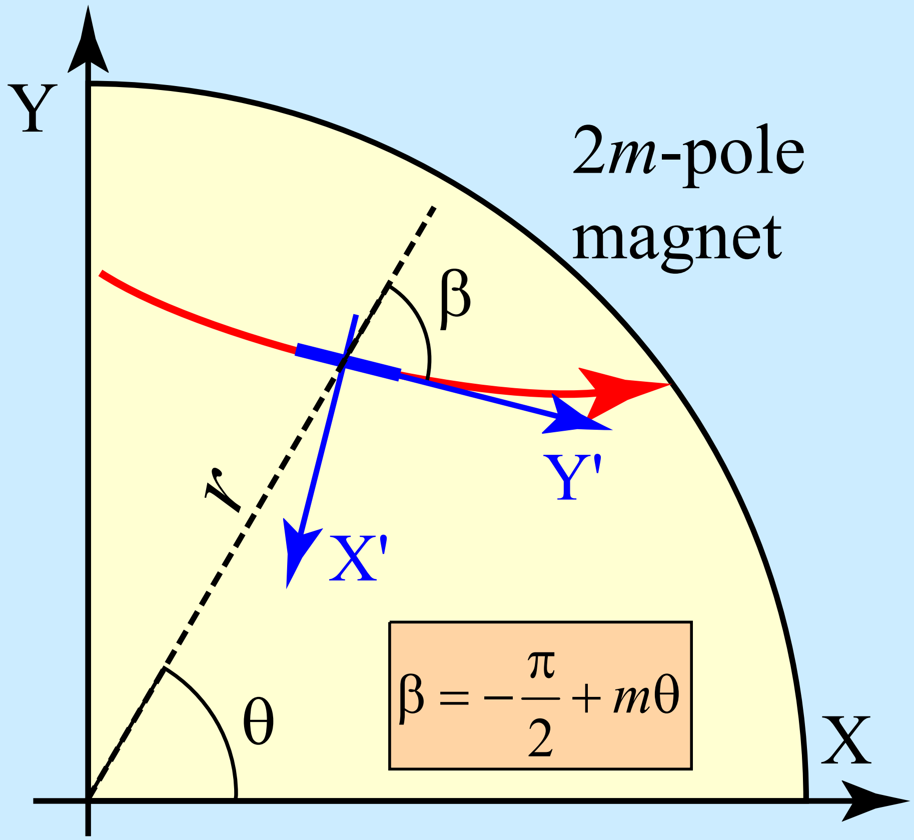
WHY EIGHT LOBES, NOT SIX ?

Colloidal Cell Theory



$$\beta = -\frac{\pi}{2} + m\theta$$

Assumptions:

- The ferrite particles are aligned with the local field direction.
- The polarizability of the particles is anisotropic.
- Polarizability along the field direction = $\alpha_{\parallel}$
- Polarizability normal to the field direction = $\alpha_{\perp}$

Colloidal Cell Theory

For observing scattered light along the direction of the incident light, the amplitude of scattered light can be shown to have the angular dependence*

$$\boxed{A(\theta) \propto \left(\alpha_{\perp} - \alpha_{\parallel}\right) \sin\{2(m-1)\theta + 2\phi\}}$$

where ϕ is the polarization angle of the incident light.

The zero intensity lines in a 2*m*-pole magnet are therefore at angles θ, given by:

$$\boxed{\theta = n\left[\frac{\pi}{2(m-1)}\right] - \left(\frac{\phi}{m-1}\right);\ \ n = 0, 1, 2, \ldots}$$

This gives 4 lobes in a quadrupole, 8 lobes in sextupole, 12 lobes in octupole, and so on. Polarization affects orientation.

* J.K. Cobb & J.J. Muray, *Nucl. Instrum. Meth.* 46 (1967) 99-105.

Limitations of Colloidal Cell

- Requires fields of moderate strength to obtain good resolution patterns. This makes the technique less suitable for warm measurements of superconducting magnets.
- Most suitable for quadrupoles. The center is not very well defined for higher order multipoles due to r^{m-1} dependence.
- The colloidal solution tends to turn cloudy with age, and may require periodical replacement.

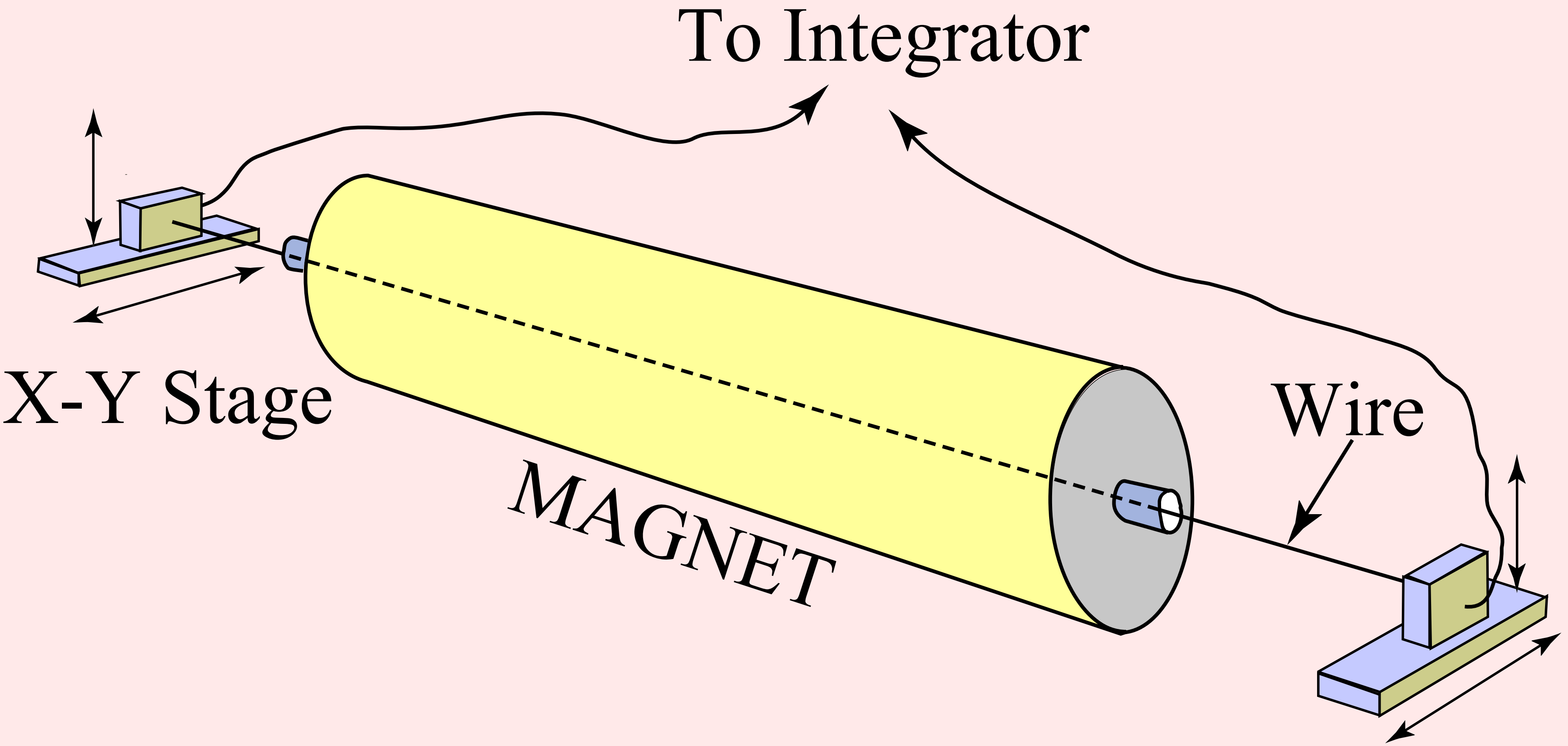
Magnetic Axis Using Field Symmetry

- Another approach to probe the field symmetry is to use a long ***stretched wire*** through the magnet bore.
- Measurements of the flux change as the wire is moved by known amounts in different directions are used to calculate the magnetic axis.
- This method gives the axis in an integral sense.
- One can also derive other quantities of interest, such as the integrated quadrupole gradient and the roll angle.

The Single Stretched Wire Technique

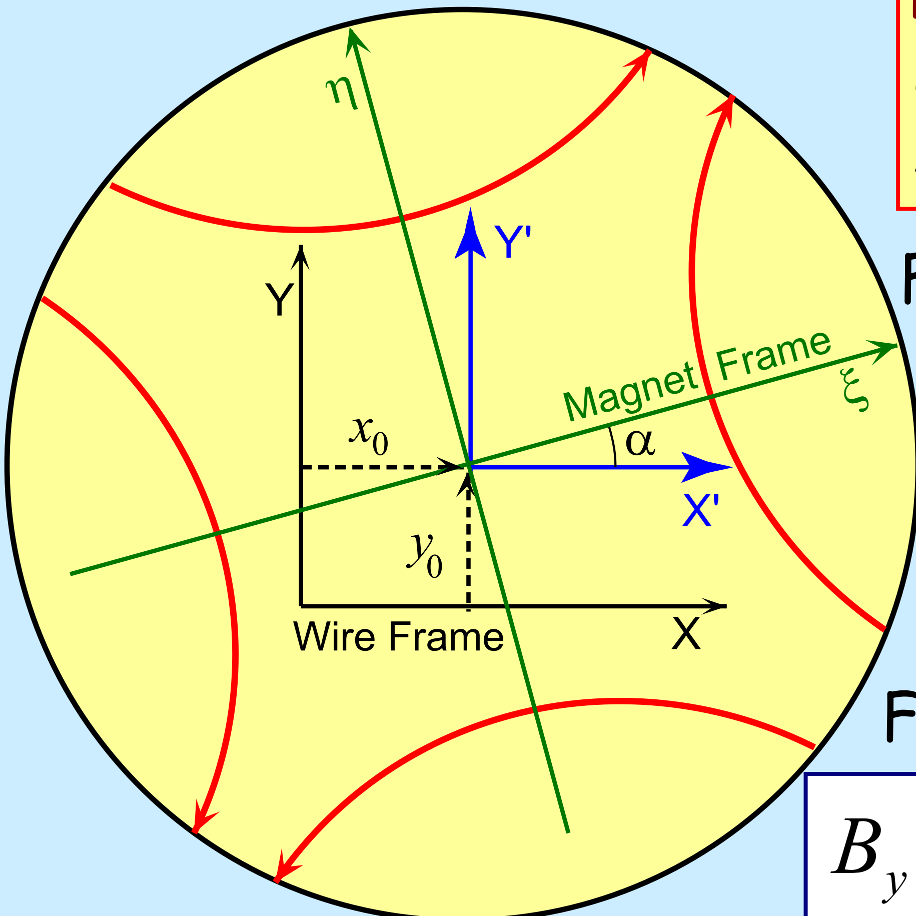
- If a stretched wire is moved in a quadrupole field, the change in flux is the same for positive and negative movements, provided the wire is initially placed at the magnetic axis. (Probing field symmetry by moving wire)
- The technique was used for measuring quadrupoles for the HERA accelerator at DESY. One also obtains the integrated gradient and the roll angle.
- The technique has been further refined at Fermilab for Main Injector and LHC IR quadrupoles.

The Single Stretched Wire Technique



Schematic of Stretched Wire Measurements

The Single Stretched Wire Technique



Pure Quadrupole field assumed

G = Gradient

$b_2 = \cos(2\alpha)$; $a_2 = -\sin(2\alpha)$

Field components in X'-Y' frame:

$$B_{y'} = G(b_2 x' - a_2 y')$$

$$B_{x'} = G(b_2 y' + a_2 x')$$

Field components in Wire frame:

$$B_y = G\left[b_2(x - x_0) - a_2(y - y_0)\right]$$

$$B_x = G\left[b_2(y - y_0) + a_2(x - x_0)\right]$$

Stretched Wire: Integral Gradient

Change in flux for a horizontal wire motion from X=0 to ±D:

$$\Phi_H^{\pm} = L_m \int_0^{\pm D} B_y \cdot dx = L_m G \left[b_2 \frac{D^2}{2} \mp (b_2 x_0 + a_2 y - a_2 y_0) D \right]$$

L_m = Magnetic Length

Similarly, for a vertical wire motion from Y = 0 to Y = ±D:

$$\Phi_V^{\pm} = L_m \int_0^{\pm D} B_x \cdot dy = L_m G \left[b_2 \frac{D^2}{2} \mp (b_2 y_0 - a_2 x + a_2 x_0) D \right]$$

The integrated gradient, $L_m G$, can be obtained from:

$$L_m G = \left(\frac{\Phi_H^{+} + \Phi_H^{-}}{b_2 D^2} \right) = \left(\frac{\Phi_V^{+} + \Phi_V^{-}}{b_2 D^2} \right)$$

For roll angles, α, less than 7 mrad, $b_2 \approx 1$ may be used with < 0.01% error.

Stretched Wire: Roll Angle

Define quantities x_0' and y_0' :

$$x_0' = -\left(\frac{D}{2}\right)\left(\frac{\Phi_H^+ - \Phi_H^-}{\Phi_H^+ + \Phi_H^-}\right); \quad y_0' = -\left(\frac{D}{2}\right)\left(\frac{\Phi_V^+ - \Phi_V^-}{\Phi_V^+ + \Phi_V^-}\right)$$

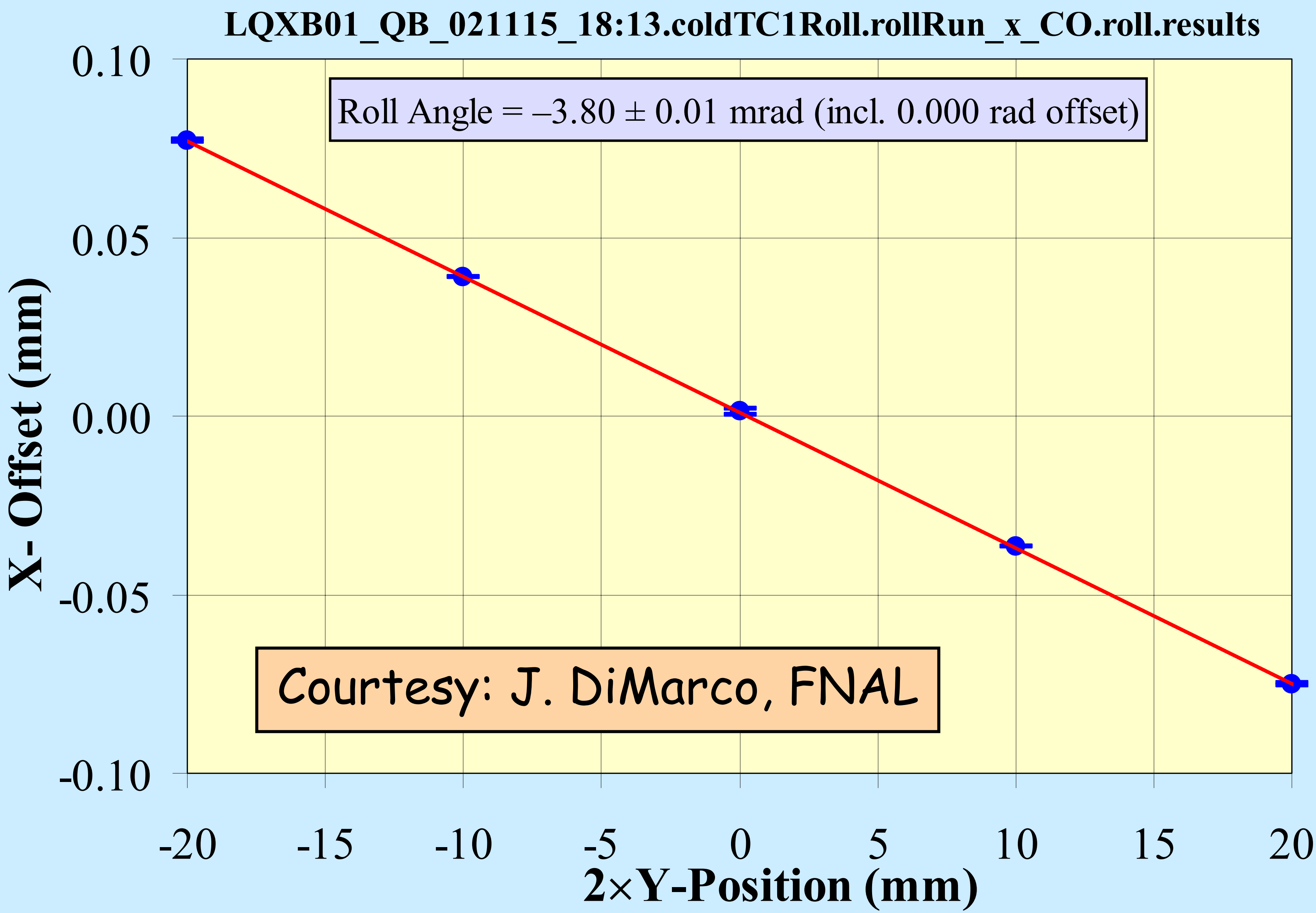
Substituting expressions for various flux changes:

$$x_0' = -\tan(2\alpha)\,y + \left[x_0 + y_0 \tan(2\alpha)\right]$$
$$y_0' = \tan(2\alpha)\,x + \left[y_0 - x_0 \tan(2\alpha)\right]$$

x_0, y_0, α are constants. x (or y) is the position where vertical (or horiz.) motion is carried out.

The roll angle, α, can be obtained by measuring the quantity x_0' as a function of y (or y_0' as a function of x) and then fitting a straight line to the data.

Roll Angle Using Stretched Wire



Stretched Wire: Magnetic Axis

Define quantities x_0' and y_0' :

$$x_0' = -\left(\frac{D}{2}\right)\left(\frac{\Phi_H^+ - \Phi_H^-}{\Phi_H^+ + \Phi_H^-}\right) = -\tan(2\alpha)y + [x_0 + y_0 \tan(2\alpha)]$$

$$y_0' = -\left(\frac{D}{2}\right)\left(\frac{\Phi_V^+ - \Phi_V^-}{\Phi_V^+ + \Phi_V^-}\right) = \tan(2\alpha)x + [y_0 - x_0 \tan(2\alpha)]$$

For horizontal motion at Y=0, or vertical motion at X=0:

$$x_0'\Big|_{y=0} = x_0 + y_0 \tan(2\alpha) \approx x_0$$

$$y_0'\Big|_{x=0} = y_0 - x_0 \tan(2\alpha) \approx y_0$$

if x_0, y_0 < 1 mm and α < 5 mrad, the error due to 2nd term is less than 1 μm.

One can use the measured value of roll angle to obtain a more precise location of the axis, if necessary.

Stretched Wire: Corrections

(1) Correction may be needed for sag of the wire:

$$\Delta y \approx \frac{wL_{wire}^2}{8T}$$

w = weight of wire per unit length
T = Tension in the wire

(2) Wire susceptibility produces force per unit length:

$$f_x = \chi G^2 A \cdot x / \mu_0$$
$$f_y = \chi G^2 A \cdot y / \mu_0$$

χ = Susceptibility; G = Gradient
A = cross sectional area of wire
(x,y) wire position from center

These effects may be compensated for by carrying out measurements as a function of tension (or equivalently, as a function of wire resonant frequency), and extrapolating to infinite tension.

Stretched Wire: "True" Axis

The treatment so far assumed that the wire is placed parallel to the magnetic axis. In general, the wire may have different offsets at different ends of the magnet.

The "True" axis may be determined by making additional measurements where the wire is moved in opposite directions at the two ends.

Such "counter-directional" measurements can also be used to locate the axial center of the magnet.

J. DiMarco et al., *Field Alignment of Quadrupole Magnets for the LHC Interaction Regions*, Proc. MT-16, Ponte Vedra Beach, Florida, Sept. 26-Oct.2, 1999, p. 127-130.

Stretched Wire: Other Options

- **AC Measurements (for Low Fields):**
  - Keep wire stationary; power magnet at ~few Hz
  - FFT of the periodic flux and current patterns
  - Measure at positive and negative wire positions

  $\Rightarrow$ *Same concept as Harmonic Antenna*

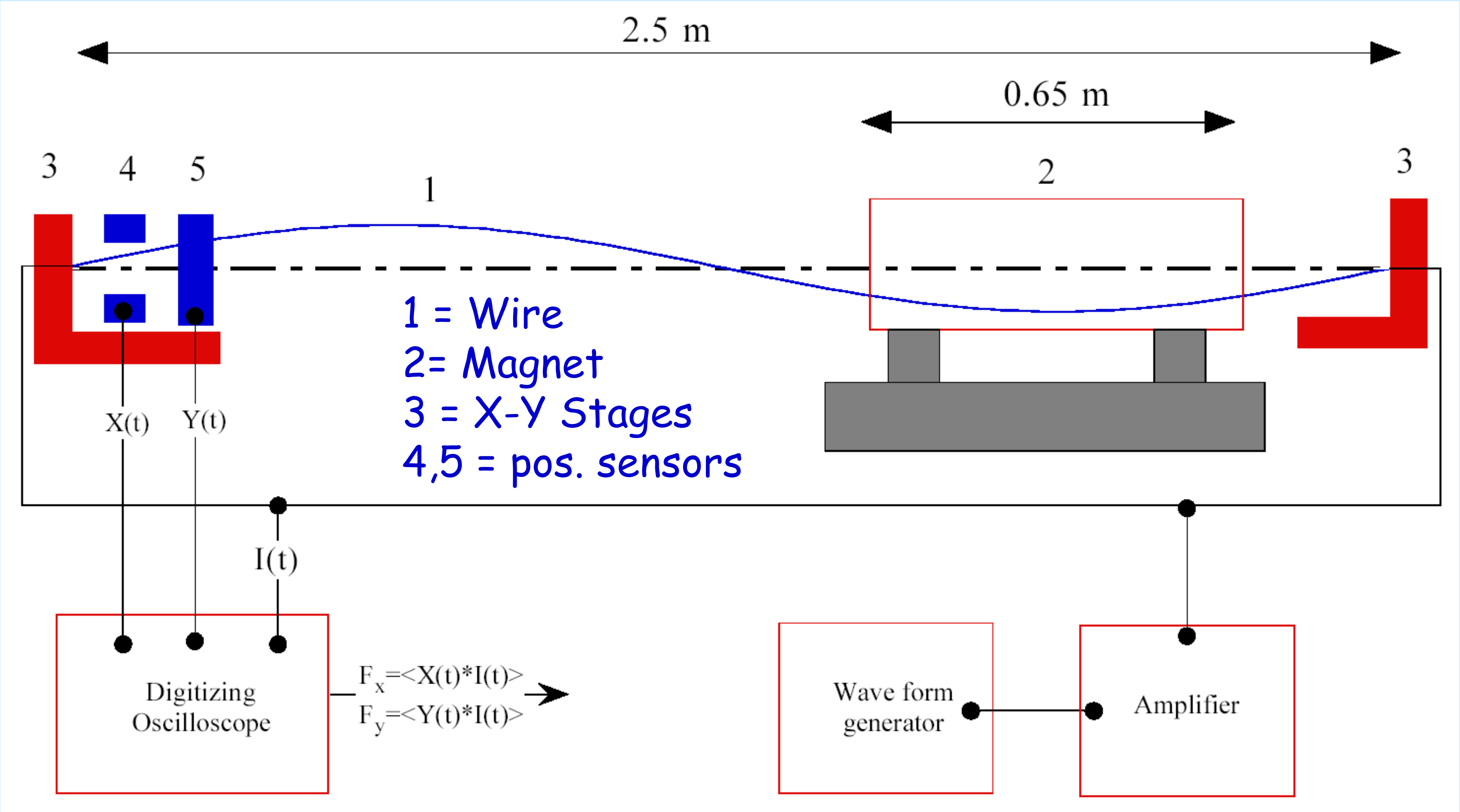
- **Rotating Stretched Wire (for Higher Multipoles):**
  - Move stretched wire along a circle
  - Measure flux change as a function of angle
  - Same hardware/software for all magnet types.

  $\Rightarrow$ *Same concept as a rotating coil*

A *Different* Way to Use a Wire

- Another approach to using a wire for magnetic axis determination is the ***Vibrating Wire Technique***.
- In this technique, an AC current is passed through a wire stretched axially in the magnet.
- If the wire is placed off-axis, the transverse fields in the quadrupole exert a periodic force on the wire, thus exciting the normal modes of vibration.
- The vibration amplitudes are studied as a function of wire position to determine the magnetic axis.
- Vibration amplitudes measured at many resonant frequencies can also give axial profile.

Vibrating Wire Measurements



from: A. Temnykh, Cornell Report CBN-99-22

Summary

- Measurements of magnetic axis are important for proper installation and alignment of magnets.
- Several techniques exist to measure magnetic axis of multipole magnets (*quadrupole and higher*).
- Dipole axis determination is governed by definition used (*high order harmonics vs. quad configuration*)
- Some techniques rely on harmonic measurements (***rotating coils, antenna, rotating stretched wire***)
- Other techniques rely on symmetry of the field (***colloidal cell, stretched wire, vibrating wire***)
- Fiducialization is often limited by survey accuracy.

Some References for More Information

- A. K. Jain, *Harmonic Coils*, CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets, Anacapri, Italy, April 11-17, 1997; CERN Report 98-05, pp. 175-217.
- A. Jain et al., *A Survey Antenna for Determining Magnetic Center*, Proc. IMMW-X, Fermilab, Oct. 13-16, 1997.
- J. Billan, *An AC Field Static System for Measuring the Magnetic Axis of LHC Superconducting Magnets in Warm Condition*, Proc. IMMW-11, BNL, Sept. 21-24, 1999.
- J. DiMarco and J. Krzywinski, *MTF Single Stretched Wire System*, Fermilab MTF note MTF-96-0001, March 19, 1996.
- A. Temnykh, *The Magnetic Center Finding Using the Vibrating Wire Technique*, Proc. IMMW-11, BNL, Sept. 21-24, 1999; and *Apparatus for Periodic Magnetic Structure Tuning*, Proc. IMMW-12, Grenoble, Oct. 1-4, 2001 (both proceedings are available on a single CD-ROM).