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HIGH TRANSITION ENERGY MAGNET LATTICES

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Abstract

Several methods have been investigated to raise the transition energies of the TRIUMF KAON Factory accelerators, designed to provide 100 μA proton beams at 30 GeV, above their acceleration ranges. This would completely avoid beam loss at transition, which, even at the few percent level, would be unacceptable for these beam currents. The methods require a periodic perturbation in either the bending properties of the lattice or the focusing properties or both. In a combined-function lattice the perturbation is provided by clustering the magnets within each superperiod. The effects on the lattice functions are described. The long drifts resulting from the magnet distribution provide natural gaps for beam extraction and injection elements and for rf accelerating cavities.

Introduction

In present proton synchrotrons the process of crossing transition results in beam losses of typically a few percent, even when remedial measures, such as "transition jumps" are incorporated. For higher intensity synchrotrons, such as in the TRIUMF KAON Factory project[1], in which 100 μA proton beams will be accelerated to 30 GeV, such losses are unacceptable because of the severe contamination they would cause. Lattices have therefore been investigated which raise transition energy γ_t above top energy, so that it never has to be crossed. In general this requires a periodic perturbation in either the bending properties of the lattice or the focusing properties or both. This paper discusses the criteria for high γ_t and their application to both combined- and separated-function magnet lattices.

An early example of a high-γ_t design was the Serpukhov proposal[2] with reversed-curvature dipoles. A less extreme approach is to use missing magnet cells, as in Saturne II[3], where there is a completely regular focusing structure. A different approach, which has been described by Teng[4,5] and Ohnuma[6], is to use pairs of trim-quadrupoles with opposite polarity in each superperiod. A related scheme has been used by Hardt[7]. For the TRIUMF application the combined-function lattices which have been considered for obtaining high γ_t are of the "modulated drift" variety, in which a superperiodicity is created by varying the drift lengths (and hence cell lengths) in a regular way, creating magnet clustering. These lattices are fully described below. Similar lattices have been adopted for the LAMPF II Booster[8]. The TRIUMF separated-function designs also achieve high γ_t through dipole magnet clustering, but through the use of missing magnet cells in a completely regular FODO quadrupole lattice[9].

Theory

The factors which are important in obtaining lattices with high transition energy may be seen by examining expressions given by Courant and Snyder[10]. The dispersion function (η_x) satisfying the inhomogeneous Hill's equation may be written in terms of Fourier components of the lattice functions:

$$\eta_x = \beta^{1/2}\,\nu^2 \sum_n \frac{a_n\, e^{in\phi}}{\nu^2-n^2}\,, \qquad a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\beta^{3/2}}{\rho}\, e^{-in\phi}\, d\phi \tag{1}$$

where ν is the horizontal tune, β the horizontal beta-function, ρ the bending radius, $\phi = \mu/\nu$ the normalized phase advance, μ the phase advance, and n an integer. As a result the momentum compaction factor is given by:

$$\frac{1}{\gamma_t^2} \equiv \alpha = \frac{\nu^3}{R} \sum_n \frac{|a_n^2|}{\nu^2-n^2} \tag{2}$$

where R is the average radius and where the summation is from $-\infty$ to $+\infty$. In many synchrotron designs the dominant term is the one with n = 0, in which case $\alpha = \alpha_o = 1/\nu^2$ and $\gamma_t \approx \nu$ (β being replaced by its average value R/ν). If instead there is an additional major contribution from the harmonic n = S (and also from n = -S) then

$$\alpha \approx \alpha_o \left(1 + 2|a_S^2|\, \frac{\nu^3}{R}\, \frac{\nu^2}{\nu^2-S^2}\right), \tag{3}$$

neglecting the contributions from other harmonics. The factor 2 takes care of the contributions from a_S and a_{-S}.

The basic principle for changing the transition energy is thus to create a harmonic component a_S by introducing a superperiodicity S. A small value of a_S will then give a large change in γ_t from the unperturbed value $\gamma_t \approx \nu$, if S is close to ν. This implies that the phase advance of each superperiod is close to 2π. To increase γ_t, S should be just above ν. If $\nu < S$ and the harmonic is strong enough then the momentum compaction factor may even become negative, taking γ_t to an imaginary value.

It may be seen from Eq. (1) that in order to generate an additional harmonic coefficient a_S, one has to modulate either β (the focusing properties) or $1/\rho$ (the bending properties) or both. The natural periodicity present due to the cell structure does not contribute significantly to γ_t since the tune of the machine is normally much less than the number of cells.

Superperiodicity in the field gradients

Courant and Snyder show that changes k(s) in the field gradient around the machine (path variable s) will give rise to a fractional change in $\beta(s)$ given by

$$\frac{\Delta\beta}{\beta} = -\frac{\nu}{\pi} \sum_{n=-\infty}^{\infty} \frac{J_n\, e^{in\phi}}{4\nu^2 - n^2} \tag{4}$$

where J_n is defined by an integral over the entire circumference C,

$$J_n = \int_0^C \beta_o(s)\, k(s)\, e^{-in\phi}\, ds\ , \tag{5}$$

$\beta_o(s)$ being the unperturbed betafunction. If the changes have superperiodicity S, the major contribution to (4) will come from terms where n = qS with integer q. Equation (1) may now be used to obtain a_S, expanding β in terms of β_o and $\Delta\beta/\beta$, and replacing $\beta_o^{3/2}/\rho$ by its average value:

$$a_S \approx \frac{3\nu}{2\pi}\left(\frac{R}{\nu}\right)^{1/2} \frac{J_S}{4\nu^2-S^2}\ . \tag{6}$$

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Since the change in α depends on a_S^2, this suggests that the harmonic $S \approx 2\nu$ might be even more effective than $S \approx \nu$. From (4), however, we note that $S \approx 2\nu$ would give rise to large and undesirable changes in $\beta(s)$. With $S \approx \nu$ it is possible to combine a large a_S with a small $\Delta\beta/\beta$, provided J_{2S} is kept small - i.e. $\beta_o(s)k(s)$ should have a significant Sth harmonic, but a negligible 2Sth harmonic.

A convenient measure to estimate the increase in the peak values of lattice functions is obtained when the magnitude of the harmonic is adjusted to bring α to zero:

$$a_S = \frac{1}{\nu^2}\sqrt{\frac{R}{\nu}\,\frac{(\nu^2-S^2)}{2}}\ . \tag{7}$$

Comparing this with (6) provides an expression for $J_S(S,\nu)$. Assuming that this is the only significant harmonic present (in particular $J_{2S} \approx 0$) one obtains:

$$\frac{\Delta\beta}{\beta} \approx \frac{4}{3}\left|1 - \frac{S}{\nu}\right|^{1/2} e^{iS\phi} \tag{8}$$

confirming that modulations in β are small for $\nu \approx S$.

Trim quadrupoles

In this method a focusing and a defocusing quadrupole, about π in phase apart, are used in each superperiod to modulate the beta function. A detailed analysis has been given by Teng[4,5] and Ohnuma[6] and also by Hardt[7] using a somewhat different approach. The method has also been used in the proposed lattice for SIS[12]. Unequal excitation of the trim quadrupoles can be used to eliminate the J_{2S} component when present. If the strengths of these two quadrupoles are $k + \delta k$ and $-k + \delta k$ then k contributes mainly to J_S and δk to J_{2S}. The magnitude of δk may then be used as a fitting variable to keep β almost unchanged at the point exactly in between the two quadrupoles.

Missing magnet lattice

A harmonic component a_S is generated in a missing magnet lattice through the variation in $1/\rho$. We consider an example in which a superperiod has p cells, e of which are empty and p-e filled with dipole magnets, in order to obtain approximate expressions for γ_t and peak η_x. For S superperiods with reflection symmetry in each :

$$a_S \approx \frac{S}{\pi}\left(\frac{R}{\nu}\right)^{3/2}\int_0^{\pi/S}\frac{1}{\rho}\cos S\phi\, d\phi \approx -\frac{\sin(\pi e/p)}{\pi}\,\frac{\sqrt{R}}{\nu^3}\,\frac{p}{p-e}$$

which gives :

$$\frac{1}{\gamma_t^2} \approx \frac{1}{\nu^2}\left[1 + 2\,\frac{\sin^2(\pi e/p)}{\pi^2}\left(\frac{p}{p-e}\right)^2\frac{\nu^2}{\nu^2-S^2}\right]\ ,$$
$$\eta_x(\phi) = \sqrt{\frac{\beta R}{\nu^3}}\left[1 - 2\,\frac{\sin(\pi e/p)}{\pi}\left(\frac{p}{p-e}\right)\frac{\nu^2}{\nu^2-S^2}\cos S\phi\right]\ . \tag{9}$$

Figure 1 gives a comparison between values obtained by tracking with the DIMAT[11] code and those obtained using Eq. (9) for a tightly filled lattice with one out of four cells in each superperiod empty. It is seen that good numbers can be obtained without knowing detailed lattice properties.

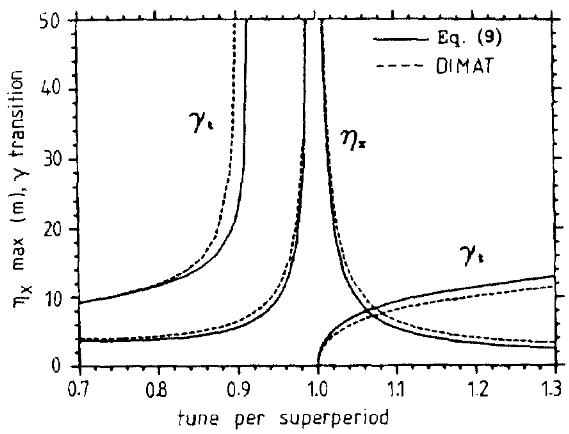

Fig. 1. Peak dispersion and γ_t as a function of tune in a missing magnet lattice.

Modulated drift lattice

In this section we describe a combined-function lattice in which variations in both β and $1/\rho$ are created by modulating the drift spaces in each superperiod, i.e. by clustering the magnets with more around the centre point of the superperiod than at the ends. This is done in such a way that the effects of both modulations enhance each other to yield a strong Sth harmonic component. In particular we describe a 30 GeV lattice with 48 cells, 12 superperiods and R = 131.6 m.

For definiteness we consider the CERN ISR magnets. These are divided in two blocks each 2.5 m long, and have field indices of -229.4 and +218.7. Figure 2 shows how the regular lattice is changed. A and B are points of reflection symmetry.

In order to obtain a high harmonic coefficient $|a_S|$, both $\beta^{3/2}$ and $1/\rho$ should be high around B and low around A (or vice versa). Decreasing the length of drifts d increases the magnet density around B, making $1/\rho$ higher there. It also increases β around B, provided the symmetry points are defined in the middle of focusing magnets. Defocusing magnets at the symmetry points will change $\beta^{3/2}$ and $1/\rho$ in opposite senses.

The four drifts a, b, c and d have to be optimized in such a way that the betafunctions do not increase too much, due for instance to a 2Sth harmonic. Also tune changes are constrained by the requirement for high γ_t. With fixed magnet parameters, as in the case studied, the magnet shifts satisfying these conditions are somewhat limited; with freedom to vary the field gradients, there is more flexibility.

Possible combinations of the four drifts "a, b, c, d" are of the type LSLS, LLSS and LMMS, where S, M and L stand for short, medium and long drifts, M being taken equal to the drift length of the regular lattice. The variation of γ_t, η_x and ν_x with the difference between long and short drifts (a - d) in these patterns is shown in Fig. 3. The crucial factor here, because of the factor $1/(\nu^2-S^2)$ in the expressions for γ_t and η_x, is the variation of ν_x with (a - d), especially where ν is close to S. The decrease in ν with increasing (a - d) in pattern LSLS becomes so large that despite an increasing $|a_S|$, η_x eventually decreases. However, γ_t continues to increase because it depends on $1/(\nu^2-S^2)$ multiplied by $|a_S^2|$. In Fig. 4 the peak values of β_x and β_y are plotted against (a - d) for these patterns.

For the 30 GeV ring structure LSLS is preferred because (a) it gives 2 long drifts in every half superperiod (24 in full machine) (b) it has the lowest $\hat{\beta}_x$ and $\hat{\eta}_x$. The harmonic modulation in $1/\rho$ for the structure LSLS can be further augmented by increasing the first long drift a and decreasing another long drift c. The effect of this variation on various parameters is shown in Fig. 5. The optimum lattice is shown in Fig.

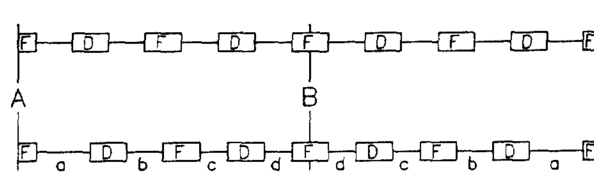

Fig. 2. Magnet rearrangement.

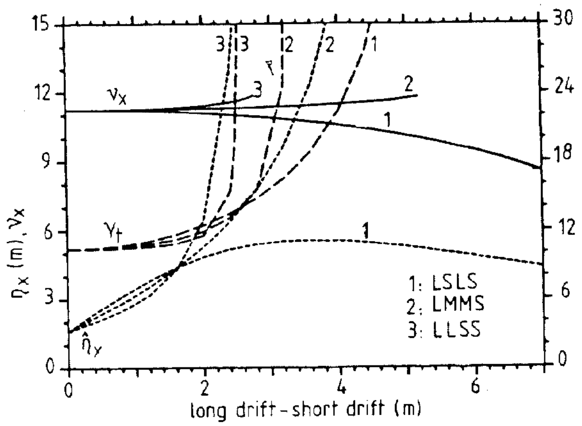

Fig. 3. Effect of increasing difference between long and short drifts.

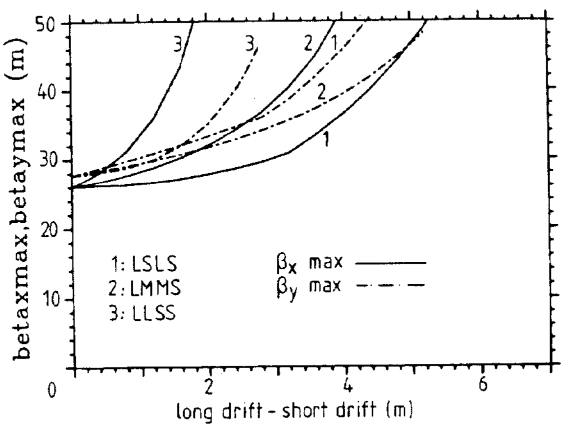

Fig. 4. Betafunction peak values versus long-short drift difference.

6 and the lattice functions are plotted in Fig. 7. The drifts a, b, c, d are 5.63, 2.34, 4.64 and 1.55 m long.

Conclusion

Various methods for obtaining high transition energy magnet lattices have been described.

The modulated drift lattice with combined-function magnets obtains an Sth harmonic dispersion from both the bending and the focusing distributions; it provides long drift spaces suitable for rf cavities and extraction and injection systems.

Acknowledgements

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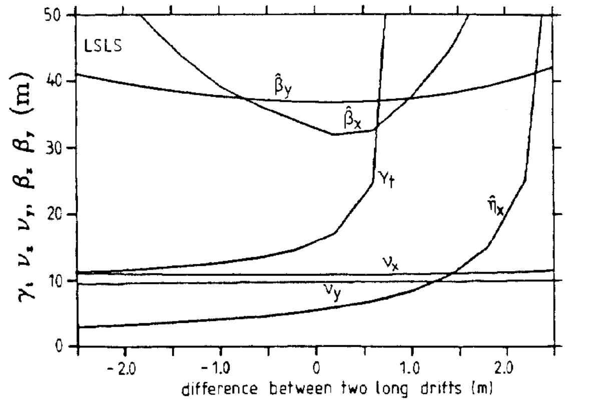

Fig. 5. Effect of increased difference in long drifts a and c with short drifts b and d fixed.

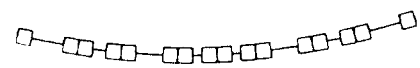

Fig. 6. Layout of one superperiod.

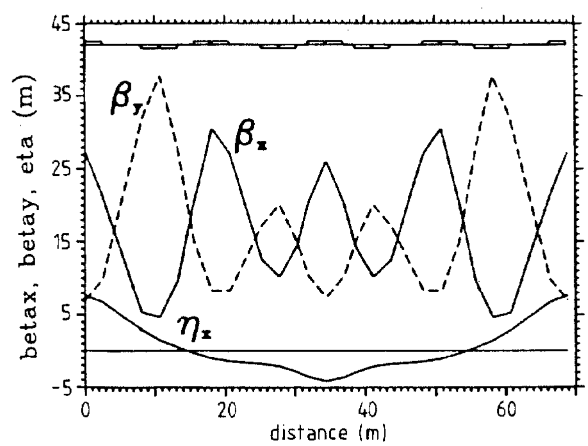

Fig. 7. Lattice functions for optimized lattice.

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