SUBJECT Straight Section Designs

1. Introduction:

Straight Section insertions are required e.g. for RF acceleration, injection and extraction. Ideally they should be designed in such a way that when they are put in the lattice the lattice functions in the original lattice are not affected.

For the TRIUMF Kaon Factory 30 GeV Syncrotron[1] four straight sections (SS) are being considered. Various kinds of SS have been designed for this purpose (several examples are given in Appendix A). These can be broadly divided in two major classes:

(a) The phase advance is adjusted so that the matrix of the insertion becomes 3x3 unit matrix ("2π insertions") or 2x2 negative unit matrix ("π insertions").

(b) The beta and alpha functions of the insertion at the end points are matched with those at the break point of the lattice.

Both types of insertions leave the beta functions in the original lattice unchanged. The Simple Collins insertions[2] (discussed in section 4) can be considered as an intermediate stage. In all but the 2π SS, the dispersion function (η) and its derivative (η') do not match. In an ideal case η and η' would be brought to zero by using dispersion suppressors[3] before the SS so that they don't get modified in the lattice. In lattices with the transition energy above the top energy, realized by a modification of the η function, special care with the SS design has to be taken if dispersion suppressors are not used.

Any Straight Section will be composed of quadrupoles and drift spaces. There is some flexibility in the way a proper combination is obtained. For example, one can either have quadrupoles at the ends and a long free space in between or have a defocussing (focussing) quadrupole in the middle, focussing (defocussing) quadrupoles in both ends and two long drifts in between.

To start with the design, rough (initial) parameters were obtained by analytic formulas (Appendix B) or simply by guess. These parameters may include the phase advance (μ), β and α. The final parameters for matching were obtained by various fitting procedures. The fitting routines usually give absurd results, or don't succeed, if the initial parameters are too far off. The computer code DIMAT[4] (with the fitting routines Simple Fit and Least Square Fit) was used for designing the Straight Sections.

2. π and 2π Sections:

In principle 2π insertions are the best since by the definition of unit matrix β and η both are matched. However they turned out to be too long for us. In the 30 GeV lattice we found them to be about 100 meters long.

π insertions which matches β only, on the other hand, could be made about 50 meters long and might be considered as a possible candidate. Some of the structures considered are:

i)	FS	QF	QD	FL	QF	QD	FS						
ii)	FS	QF	QD	FL	QD	QF	FS						
iii)	FS	QF	FL	QD	QD	FL	QF	FS					
iv)	QF	FS	QD	FL	QD	FS	QF	QF	FS	QD	FL	QD	FS QF.

Here FS and FL stands for short and long drift spaces. QF and QD are horizontally focussing and defocussing quads. Note that all but (i) have mirror symmetry.

3. Straight Sections with Lattice Functions Matching:

In this class β and α at the end points of the insertion are matched with those at the break point of the regular lattice. In all cases considered the lattice was broken at the point of reflection symmetry where α goes to zero. This occurs between two magnet blocks and at two places in our superperiod.

The Straight Sections were also designed to have reflection symmetry so that α is zero at the end points. Therefore in this approach we only needed to match β_x and β_y. As mentioned before η will not be matched.

This class of SS was given maximum consideration and several types with various lengths, phase advance and beta functions were designed. Some of the structures considered are:

v)	FS	QF	QD	FL	QD	QF	FS	
vi)	QF	FS	QD	FL	QD	FS	QF	
vii)	FS	QF	FL	QD	QD	FL	QF	FS

The symbols have been explained in the previous section.

4. Collins Insertions:

Simple Collins insertions[2),5)] (with an F and a D quad and a long drift in between - phase advance being $\pi/2$ for the maximum drift length) were also designed. However the requirements of the lattice functions matching $\alpha_x = -\alpha_y$ and $\gamma_x = \gamma_y$ (or $\beta_x = \beta_y$ since $\gamma = (1 + \alpha^2)/\beta$), were never exactly met. The mismatch will not be large if β_x and β_y are not too different at the point where $\alpha_x = -\alpha_y$.

The structure of Simple Collins insertion is:

FS QF FL QD FS.

5. Transformers:

If the lattice functions at the end points of SS are not matched with those at the break point of the normal lattice then transformers are required to do this matching. The values of lattice functions at the two ends of transformers are different. At the one end it is made as that of the break point b of the normal lattice and at the other end as that of the end point s of SS. Therefore they equivalently transform the lattice functions of point s to that of the point b.

If ${}_bT_s$ and ${}_sT_b$ symbolize the two transformers (with the lower script meaning that at that end they have taken the values of the lattice functions of the point indicated) and similarly if ${}_sS_s$ symbolizes SS, then the completely matched structure would like

$$ {}_bT_s \quad {}_sS_s \quad {}_sT_b . $$

However, in our case we avoided the need of transformers by making the lattice functions of point s equal to those at the point b at the first step itself.

Acknowledgement:

We appreciate the help and guidance from R. Servancks† in the design of the Straight Sections.

References:

1. Progress Report on the feasibility of using the ISR magnets in a TRIUMF Kaon factory, June 1983.

2. T. Collins; CEA - 86 (1961).

3. J.I.M. Botman, Dispersion Suppressors, TRI-DN-83-33.

4. R. Servanckx, Computer program DIMAT for designing the particle accelerators.

5. E. Keil, CERN Report 77-13, page 26.

6. G.M. Stinson, A course in Beam Optics, TRI-DNA-83-9.

7. A.A. Garren, Design of long straight sections for synchrotrons, V International Conference on high energy accelerators, Frascati (1965), page 22.

†)Univ. of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 0W0.

APPENDIX A

Here we mention the parameters of few selected Straight Sections.

1. **π Section**

Structure: QF FS QD FL QD FS QF QF FS QD FL QD FS QF
Lengths: FL = 15 meters, FS = 1 meter, QF = 1.5 meters, QD = 1.5 meters.
Total length: 46 meters.
Strength of quads: QF = .0987107 m^{-1}, QD = −.1075857 m^{-1}.
Phase advance: $\mu_x = \mu_y = \pi$.

2. **Collins Insertion**

Structure: FS QF FL QD FS
Lengths: FL = 20 meters, FS = .5 meters, QF = 2 meters, QD = 2 meters.
Total length: 25 meters.
Strength of quads: QF = −QD = .10073288 meter^{-1}.
Phase advance: $\mu_x = \mu_y = \pi/2$.
$\beta_x = \beta_y$ = 23.675 meters and $\alpha_x = -\alpha_y = -4.3784$.

3. **Straight Sections having reflection symmetry**

Since they have reflection symmetry $\alpha_x = \alpha_y = 0$ always.

a) Structure: FS QF FL QD QD FL QF FS
Lengths: FL = 4.0 meters, FS = 1.5 meters, QF = .75 meters, QD = .75 meters.
Total length: 14 meters.
Strength of quads: QF – .09221 meter^{-1}, QD = −.09021 meter^{-1}.
Phase advance: $\mu_x - \mu_y = .125$
β_x = 23.544 meters, β_y = 13.653 meters.

b) Structure: QF FS QD FL QD FS QF
Lengths: FL = 8 meters, FS = 1 meter, QF = 1 meter, QD = 1 meter.
Total length: 14 meters.
Strength of quads: QF = .12256 meter^{-1}, QD = −.1193 meter^{-1}
Phase advance: $\mu_x = .154$, $\mu_y = .116$.
β_x = 23.544 meters, β_y = 13.652 meters.

c) Structure: QD FS QF FL QF FS QD
Lengths: FL = 15 meters, FS = 1 meter, QF = 1.3 meters, QD = 1.2 meters.
Total length: 22 meters.
Strength of quads: QF = .10504 meter^{-1}, QD = .11575 meter^{-1}.
Phase advance: $\mu_x = .177$, $\mu_y = .247$
β_x = 13.094 meters, β_y = 34.169 meters.

APPENDIX B

In this appendix we mention a few analytic formulas and results which were very frequently used.

a) If the SS has mirror symmetry and R is the transfer matrix of first half of it, the transfer matrix of the full SS is obtained by

$$S = \begin{pmatrix} 2R_{11} R_{22}-1 & 2R_{12} R_{22} \\ 2R_{11} R_{21} & 2R_{12} R_{22}-1 \end{pmatrix}$$

$S_{11} = S_{22}$ means $\alpha = 0$ always.

Phase advance and β function can be obtained by using definitions of twiss matrix

$$T = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Therefore $\mu = \cos^{-1}(2R_{11}R_{22}-1)$,

and $\beta = R_{12}/\mathrm{SQRT}(R_{11}/R_{22} - R_{11}^2)$.

b) The R matrix of one half of the mirror symmetric SS is given below in two cases.

i) Structure: $\underbrace{\text{F1 QF QD F2}}_{R}$ F2 QD QF F1

Lengths: $F1 = \ell_1$, $F2 = \ell_2$, $QF = m_1$, $QD = m_2$
Strength of quads: $QF = k_1$, $QD = k_2$
and $\theta_1 = k_1 m_1$, $\theta_2 = k_2 m_2$

Horizontal matrix elements

$$R_{11} = \cos\theta_1 (\cosh\theta_2 + k_2 \ell_2 \sinh\theta_2) - k_1 \sin\theta_1 (1/k_2 \sinh\theta_2 + \ell_2 \cosh\theta_2)$$

$$R_{12} = (\ell_1 \cos\theta_1 + 1/k_1 \sin\theta_1)(\cosh\theta_2 + k_2\ell_2 \sinh\theta_2) + (-k_1\ell_1 \sin\theta_1 + \cos\theta_1)(1/k_2 \sinh\theta_2 + \ell_2 \cosh\theta_2)$$

$$R_{21} = k_2 \cos\theta_1 \sinh\theta_2 - k_1 \sin\theta_1 \cosh\theta_2$$

$$R_{22} = k_2 (\ell_1 \cos\theta_1 + 1/k_1 \sin\theta_1) \sinh\theta_2 + (-k_1\ell_1 \sin\theta_1 + \cos\theta_1) \cosh\theta_2$$

And similarly in the vertical plane.

ii) Structure: $\underbrace{\text{F1 QF F2 QD}}_{R}$ QD F2 QF F1

Symbols as in part (i).

Horizontal matrix elements:

$$R_{11} = \cos\theta_1 \cosh\theta_2 - k_1 \sin\theta_1 (\ell_2 \cosh\theta_2 + 1/k_2 \sinh\theta_2)$$

$$R_{12} = (1/k_1 \sin\theta_1 + \ell_1 \cos\theta_1) \cosh\theta_2 + (\cos\theta_1 - k_1\ell_1 \sin\theta_1)$$
$$(\ell_2 \cosh\theta_2 + 1/k_2 \sinh\theta_2)$$

$$R_{21} = k_2 \cos\theta_1 \sinh\theta_2 - k_1 \sin\theta_1 (\ell_2 k_2 \sinh\theta_2 + \cosh\theta_2)$$

$$R_{22} = k_2 (1/k_1 \sin\theta_1 + \ell_1 \cos\theta_1) \sin\theta_2 + (-\ell_1 k_1 \sin\theta_1 + \cos\theta_1)$$
$$(\ell_2 k_2 \sinh\theta_2 + \cosh\theta_2)$$

And similarly in the vertical plane.