TRIUMF | 4004 WESBROOK MALL, VANCOUVER, B.C. V6T 2A3

DESIGN NOTE | NAME Ramesh C. Gupta

DATE August 1983 | FILE NO. TRI-DN-83-36

SUBJECT Methods of Designing Synchrotrons with High Transition Energy

1. Introduction:

In high current synchrotrons space charge defocussing forces and beam instabilities at the transition energy (E_t) limit the beam current. Therefore for the TRIUMF Kaon Factory the lattice is being designed in such a way that γ_t stays above the top energy of the synchrotorn.

The methods of raising γ_t 1), 2), 3), 4), 5) consists of introducing a large kth harmonic in the lattice functions with k just above ν_x or $2\nu_x$. It is usually done with the help of k pairs of quadrupoles. Another method has been developed for the proposed main synchrotron where the same has been achieved by modulating the drift spaces. In addition to raising γ_t the design also provides long drifts at certain places which may be utilized for beam extraction 6), etc.

2. Theory:

In the notation of Courant and Snyder 4) (Page 42) the transition energy is given by

$$\frac{1}{\gamma_t^2} = \frac{\nu^3}{R} \sum_{k=0}^{\infty} \frac{|a_k^2|}{\nu^2 - k^2}$$

$$\text{with} \quad a_k = \frac{1}{2\pi} \int_0^{2\pi} \frac{\beta^{3/2}}{\rho} e^{-ik\phi} d\phi,$$

and where β, ρ and R have their usual meaning. For k=0, $a_0^2 = R/\nu^3$ and when only one other harmonic (k=n) is present one obtains

$$\frac{1}{\gamma_t^2} = \frac{1}{\nu^2} \left(1 + |a_n^2| \frac{\nu^3}{R} \frac{\nu^2}{\nu^2 - n^2}\right)$$

Therefore γ_t can be changed from $\gamma_t = \nu$ by introducing a harmonic of amplitude a_n and the change will be large if ν is close to n.

3. Method:

The desired harmonic may be obtained with the help of n pairs of quads 1),2),3) or with n "compensating magnets" with reversed fields 4),5). The method developed here requires neither of them.

The first step in the methods of raising γ_t is to make a superperiod of cells with a tune close to but below an integer n (n=1 in our case) and then in the next step to introduce the nth harmonic. In our case since a basic FODO cell has a tune of about .25 we make the following superperiod with four basic cells:

FODOFODOFODOFODO (A)

The harmonic k=1 is present if the two halves of the above superperiod have different net focussing strengths with one half having above the average value and the other half below.

We suggest two new ways of doing it:

(a) By modulating the focussing strengths of the magnets in a combined function machine or equivalently by modulating the strengths of quads in a separated function machine.
(b) By modulating the spacing of the magnets.

Since at this stage we are considering to use the CERN/ISR magnets for the main synchrotron, the field index of the magnets cannot be changed. In example (i) of the next section we illustrate a case where the focussing of F magnets has been modulated by proper pairing of the magnet blocks.

Several combinations of modifying equal drifts to unequal drifts have been tried (e.g. see examples ii, iii, iv of the next section). The aim was to raise γ_t with keeping the maxima of β_x, β_y and η_x to the lowest possible value. One of such a combination is:

FODOOFOODOFOODOFODOO (B)

a b c d

where a single O means a short drift and two O's (OO) means a long drift. We have also defined the new lenses a, b, c and d by combining the F and D magnets which are separated by a short drift.

The desired harmonic has been introduced in this way because lens d is net defocussing and b is focussing with a & c having the same strength in first approximation.

The ISR magnets, both F and D, appear in two types of blocks - short blocks and long blocks - with a small difference in their focussing properties. This difference may be regarded as a modulation of the kind described in option (a). The difference may further be increased by increasing the separation between the two short blocks. Then going to an option which is hybrid of (a) and (b) more varieties to obtain a proper combination arise.

The largest possible difference between the short and the long drift was desired to obtain a long free space where septum and kicker magnets can be put for beam extraction [6].

4. Examples:

The original lattice consists of eight equal drifts of length 3.5 m with γ_t being 10.2. We present four examples in which γ_t is raised to about 33. In the first example option (a) has been used and in the last three examples option (b): with three ways of modulating the drifts. We mention that in the examples considered, all lattice have reflection symmetry in the superperiods (though not all have the point of symmetry at the same place).

(i) Since the field index of the magnets cannot be changed, the nearest equivalent of it has been done. The length of the magnet has been changed by replacing one short F block to another place in the way as indicated in Fig. 1. Now the 1:3 ratio of the lengths of the new F magnets gives the desired variation in the focussing properties. Since the magnet length ratio is strictly 1:3 one does not have much flexibility in this option.

(ii) The superperiod is made up of two halves with each half having four equal drifts. The difference between the drift lengths of two halves is raised till γ_t reaches the desired value (Fig. 3). The two drift lengths obtained are: 4.8713 m and 2.1287 m.

(iii) In the first half of the superperiod the four drifts have a successively increasing lengths and the second half is reflection symmetric of it with the four drifts having a successively decreasing lengths (Fig. 3). The precise drift lengths change according to a sine function and the obtained values are respectively :1.7592, 2.6296, 4.3704, 5.2408 in metres.

(iv) This is basically the same example as the lattice B (explained in detail in section 3). In the first half we have four drifts arranged as:
Short Long Long Short
and in the second half as:
Long Short Short Long.
The obtained value of the long drift is 5.57 m and of the short drift is 1.43 m.

5. Conclusion:

We conclude by saying that for raising the transition energy the basic design of the synchrotron itself could be made in such a way that it inherently contains the desired harmonic, rather than introducing it with the help of additional quadrupoles in an otherwise originally optimized lattice.

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6. Acknowledgments:

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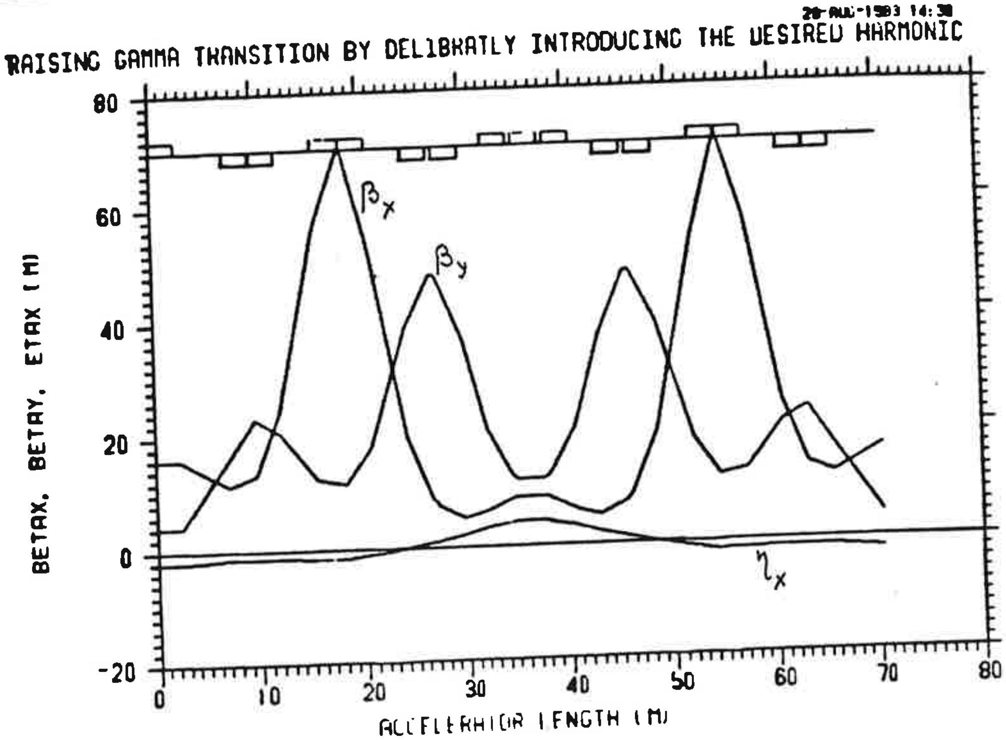

Fig. 1

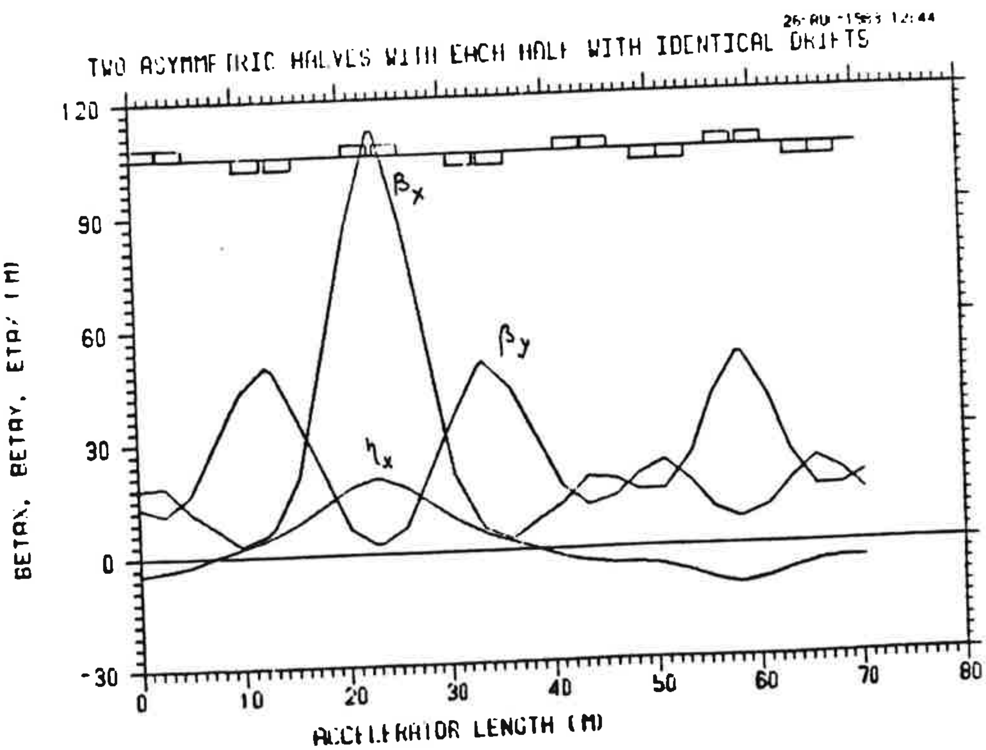

Fig. 2

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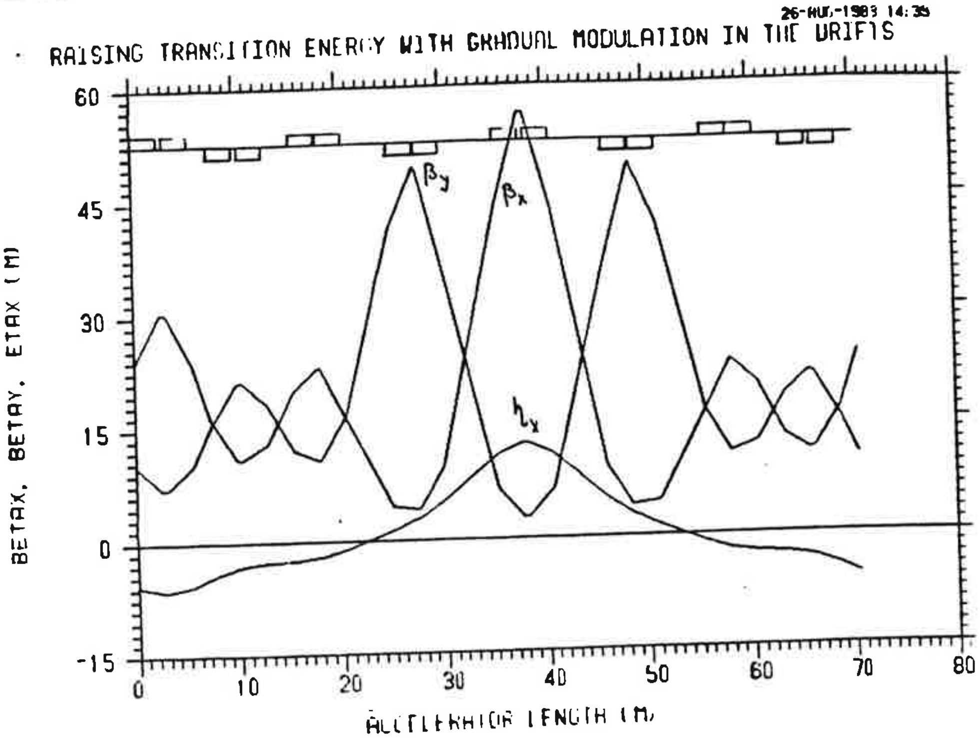

Fig. 3

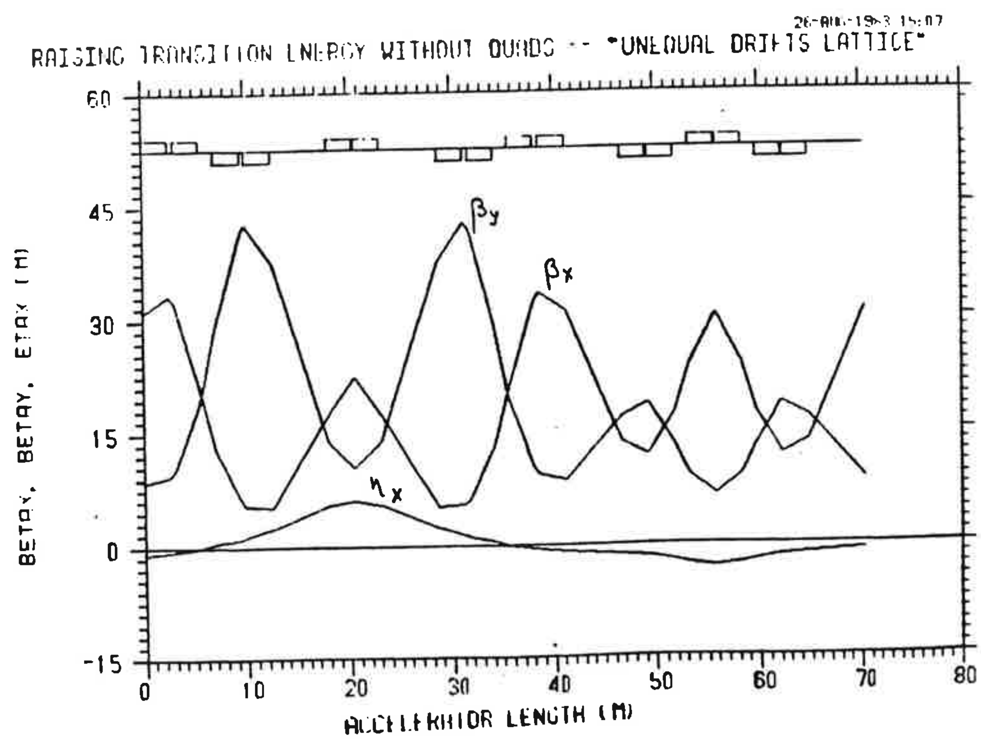

Fig. 4